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A.R.D.E. REPORT (P) 8/57

Rocket accuracy: The effect of non-rigidity and thermal forces on the dispersion of rockets

A. G. Walters
R. J. Rosser
J. W. Martin

Fort Halstead
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A. G. Walters, R. J. Rosser, J. W. Martin

(P.6)

Summary

The effect on rocket accuracy of non-rigidity, and especially of the motor bending when heated by the propellant gas, has been investigated theoretically and experimentally. The experiments were carried out with a Service rocket (Rocket Aircraft 3-inch No. 1) which was chosen as the most suitable for the purposes of the investigation. It is demonstrated how this rocket (as typical) can be modified so that this source of inaccuracy is suppressed. Charge designs in which the propellant gases have free access to the motor body wall (X11), limited access (inhibited star-section colloidal propellants), or no access until late in burning (plastic propellant) are considered. Limited access may cause as much distortion as free access.

The report describes only mathematical and laboratory work; the results of projection trials for dispersion, which should show the effect of removing this source of inaccuracy, will be dealt with in a subsequent report.

Approved for issue:-

Ewen M'Ewen, Director

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1. INTRODUCTION

Many attempts have been made in the past to calculate theoretically the dispersion of unrotated rockets. In these so called tolerance theories it is assumed that the rocket behaves as a rigid body and the dispersion is due to the measurable mechanical tolerances. The angular malalignment of the nozzle exit cone axis from the rocket axis gives theoretical trajectory deviations considerably in excess of those from the other tolerances. The emergent gas jet derives almost the whole of its momentum from the nozzle and it can therefore be assumed that the line of thrust is along the nozzle axis. If this axis is malaligned so that it does not pass through the centre of gravity of the rocket, there is a turning moment on the rocket which causes it to turn from the trajectory i.e. the rocket is yawed. The main thrust then propels the rocket away from the intended trajectory. The methods used in the calculation of these deviations are given in detail in ref. 5. For the Rocket Aircraft 3-inch No. 1, a nozzle exit cone angular malalignment of 10^{-3} radians deviates the final trajectory 0.8 degs. from the line of launch. The mean measured malalignment for a large number of these rockets is 0.5×10^{-3} radians approximately corresponding to a trajectory deviation of 0.4 degs. This is a circular distribution since the rockets are placed on the launcher with random orientation and it is multiplied by $2/\pi$ to reduce it to a linear mean deviation (i.e. deviations in one plane). Thus the expected linear mean deviation is 0.3 degs. However, the mean dispersion of this rocket when filled tubular charge as obtained from a large series of firings carried out at P.D.E. Aberporth in 1944 is 1.4 degs. This is appreciably greater than the value 0.3 degs. Similar discrepancies are noted with almost all rockets. There is little reason to doubt the fundamental accuracy of the ballistic theory employed. For example the ratios of the dispersions of the various 3-inch rockets as calculated by this method for the 12 ft., 24 ft., and 48 ft. projectors are in reasonably good agreement with the observed values, also the ratios of the dispersions with different fin sizes are given correctly. The theory also gives the correct wind constants. We can thus employ the theory with some confidence provided we know the disturbing forces.

Three fundamental assumptions are made in the tolerance theory.

1. The rocket behaves as a rigid body.

2. The resulting thrust vector arising from the emerging jet lies along the nozzle exit cone axis.

3. The rocket is launched perfectly so that the angle of the rocket axis and its angular velocity at the moment of launch are constant from rocket to rocket.

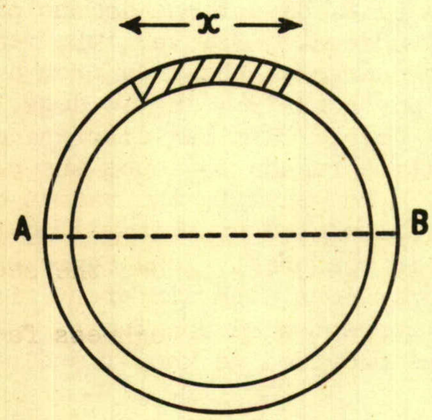
It is with the first of these assumptions that we are concerned in this report. The second and third assumptions are considered in later reports.

For the practical investigation it is necessary to choose a specific design of rocket on which to work. The design chosen is the Rocket Aircraft 3-inch No. 1. This particular rocket has been in Service since the early days of the war and it is considered to be particularly useful for the present investigation because (a) an adequate supply of components is available, (b) it has a long history of dispersion and other trials so that its behaviour is well established, (c) a variety of charge designs can be adapted to it, including designs which admit a copious flow of propellant gas against the wall of the motor body.

A detailed theory has been developed by means of which the motion of a non-rigid rocket is examined during burning. If the motor tube bends during flight, the line of the nozzle axis is displaced from the centre of gravity of the rocket thus producing a turning moment on the rocket which causes it to

yaw and deviate from the expected trajectory. The motor tube is under high internal pressure and also large set-back forces in flight. It can be shown that due to mechanical tolerances mainly in the tube wall thickness these forces bend the tube. Neglecting temperature effects and taking the Young's modulus of the steel to be that at normal temperatures, it is shown that the calculated bending due to the measured mechanical tolerances gives rise to deviations in flight which are small compared with deviations observed in trials. Thus the assumption of rigidity of the rocket tube is valid if we neglect the effect of temperature. However, it is found that the asymmetric temperature distributions around the tube circumference arising from the various factors discussed later give rise to more serious bending of the tube and it is calculated that the deviations resulting from these sources are of the same order as the deviations observed in flight. This effect is undoubtedly a major source of the dispersion of this rocket.

There are various factors which give rise to the asymmetric temperature distribution around the tube section. To illustrate the orders of magnitude, the following examples are given for the 3-inch rocket.



(i) If the upper side of the tube (i.e. the section above the plane AB) is approximately 12°C hotter than the lower side, the resultant bending of the rocket is equivalent to offsetting the nozzle axis by 10^{-3} radians thus giving a mean circular deviation in flight of 0.8° approximately.

(ii) In the diagram above the strip of breadth x ins. (x being small) has a length equal to that of the tube. If the mean temperature of this longitudinal strip differs from that of the remainder of the tube by an amount $35/x$ °C, the resultant bending is again equivalent to offsetting the nozzle axis by 10^{-3} radians giving a deviation in flight of 0.8° for the 3-inch aircraft rocket.

These temperature differences are small when compared with the temperatures attained by the tube. Some typical values of the latter for tubular SU/K charges for tubes with and without a refractory insulating coating are given in Table I, the quoted temperatures being those at the middle section of the tube.

TABLE I

Theoretical temperatures attained by the walls of the 3-inch rocket tube

Time secs.	Temperature of coated tube °C. Inner surface of coating θ_1	Inner tube surface θ_2	Outer tube surface θ_3	Temperature of uncoated tube °C. Inner tube surface θ_2	Outer tube surface θ_3
0.2	870	210	130	370	190
0.4	750	280	220	470	350
0.6	710	340	280	550	440
0.8	690	370	330	610	520

The effects are not quite as serious as they first appear; the bending calculated in (i) and (ii) is based on the assumption that the temperature differences are constant along the whole length of the tube, an unlikely condition in practice. The principal factors which cause asymmetric heating of the tube are

(i) slight asymmetries in the refractory coating thickness,

(ii) charge asymmetry, particularly lateral displacement, which leads to non-uniform gas flow across the section,

(iii) possible variations in the roughness factor of the inside of the tube,

(iv) variations in the wall thickness of the tube around the section.

The tube coating consists of 0.006 ins. refractory coating with a top protective bakelite varnish coating of 0.002 ins. On the assumption that the coating remains intact, it is seen that the temperature drop across the coating varies from approximately 900°C at t = 0.1 secs. to 300°C at 0.8 secs. That asymmetric coating thickness affects the dispersion was shown by a dispersion trial. In this trial the upper half of the tube was given a coating of thickness 0.01 ins., and the lower 0.002 ins.; the increase of dispersion was marked.

The thermal stresses in rocket motor tubes have been examined in some considerable detail by Oldroyd (Ref. 6). He has shown that in the case of the Rocket Aircraft 3-inch No. 1 filled tubular charge, there is appreciable plastic flow on the inside and outside surfaces of the tube wall due to high thermal stresses. The effects are maximum in the early stages of burning when the temperature gradients are maximum. Some evidence for this comes from the bending tests for it was noted with tubular charges that the frequency of vibration as observed on the bending records decreased markedly at 0.1 secs. after commencement of burning, and tended to recover after 0.3 secs. The reduced vibration frequency suggests reduction in the Young's Modulus of the tube and this may be expected to have an effect on the bending. The normal tubes are 0.08 ins. wall thickness. Increasing the wall thickness to 0.18 ins. markedly increases the thermal stresses due to the increase in the temperature gradient. Comparative measurements of the bending of the two sets of tubes show that the 0.08 ins. tubes bend much more than the thicker tubes and in fact the ratio is that to be expected if we assume Young's Modulus as constant. The observed bending is therefore apparently due in the main to asymmetric temperature distribution. The records of the bending tests together with the

measurements of the tubes before and after firing are being analysed statistically to determine the effect of thermal stresses and it is hoped to give the conclusions in a later report.

It should be noted that the bending observed in the experiments described in this report is not directly applicable to flight conditions. In flight, the motor tube is constrained elastically only by the shell and venturi assemblies, whereas in static firings it is clamped rigidly at the head end. In flight the natural frequency of vibration is calculated to be 87 cycles per second while in the bending tests it is 30 cycles per second, assuming in both cases the value of Young's modulus to be that at normal temperature. However, both these values are sufficiently high to justify the use of the mean bending.

In flight the charge is subjected to an additional set-back force and, in the case of the tubular charge, this probably causes relative displacement of the charge and tube as the charge is elastically an unstable strut. This displacement would have a serious effect on the bending. Cruciform charges on the other hand are forced against the tube and this may prevent gas flow between the arms of the charge and the tube so removing a serious source of bending. Thus the bending tests may not give a trustworthy comparison of the two charges in flight.

2. EXPERIMENTAL RESULTS

2.1 Instrumentation used in the measurement of bending

The experimental arrangement for measuring the amount of bending of the 3-inch rocket tube during firing is shown in Figure 30. The head end of the motor tube is clamped rigidly in two strong steel clamps, spaced 3-inches apart, the remainder of the rocket tube being unsupported so as to permit free bending. To determine the amount of bending the lateral movement of the venturi end of the rocket furthermost from the clamps is measured. For this, two experimental techniques have been used, the first based on the use of capacity gauges and the second on magslips. A considerable amount of effort was expended in an attempt to obtain reliable measurements with the capacity gauges and some details are given in order to illustrate the difficulties encountered.
Capacity gauge method Figure 31 shows the first arrangement used. A metal ring, insulated from the rocket tube by a thin layer of mica acts as one electrode. The second electrode consists of a circular disc which is fitted to a coarse micrometer screw thread. The condenser formed by these two electrodes is placed in the arms of a bridge. Movement of the rocket then varies the capacity of the electrode system and causes the balance of the bridge to alter.

The electronic circuitry associated with the bridge is shown in Figure 32. The original unit was fed with raw a.c. to the H.T. line giving an output of rectified pulses of mains frequency. Subsequently the H.T. was obtained from a stabilised D.C. power supply giving an output of radio frequency voltage.

The first valve, L63 was an oscillator supplying a 500 Kc. voltage to the bridge. This was followed by a two stage amplifier consisting of two variable mu valves (KTW61). The output from this amplifier was fed directly to the oscilloscope and camera. Movement of the rocket then altered the amplitude of the 500 Kc. sine-wave appearing on the oscilloscope. Calibration was carried out by screwing the electrode in steps of 1/10th inch from the ring mounted round the rocket. The electronic apparatus was situated in a room adjacent to the firing chamber and this particular type of bridge had the advantage that the screened leads to the gauge could be made at least 30 feet long without much decrease in sensitivity or difficulty in obtaining balance.

A certain amount of success was obtained using this apparatus, in particular the existence of bending was established. The traces were, however, susceptible to short violent kicks of a few hundredths of a second duration throughout the recording. Although there was no great difficulty in reading the records by extrapolating, the presence of these disturbances was unsatisfactory. The origin of the disturbances was the rocket itself which set up violent tremors in the neighbouring buildings, shaking the electronic instruments.

The trials with the capacity gauge were exploratory in nature and with the arrangement described above it would not be possible to measure movement in two planes simultaneously since the gauge itself must be free to 'float'. Another capacitance meter made by Messrs. Fielden's Electronics employs a separate bridge unit and a capacity gauge with one side earthed. It appeared that using the arrangement of Figure 34, bending could be recorded in two axes. An initial difficulty was that beating occurs between the two oscillators but this was overcome by disconnecting one of them and supplying both bridge units from one oscillator. The biggest difficulty, however, was that the lead from capacity gauge to bridge unit was only two feet long so that the bridges had to be placed in the actual firing chamber. The system appeared to function in principle, but once again the disturbance set up by the rocket on firing caused interference. Extension of the capacity gauge lead from the bridge unit decreased the sensitivity correspondingly, making it necessary to raise the amplifier gain which in turn raised the sensitivity of the recording apparatus to vibration and shock.

Use of Magslips

A much more robust recording system was sought and the use of magslips appeared worthy of trial. These devices are usually found as position indicators in servo-mechanisms. It was soon apparent that considerable advantages were associated with their use. In particular the output-displacement characteristic was sensibly linear and their sensitivity high, so that no auxiliary amplifier other than that built into the oscilloscope was needed. The operating frequency was 1 Kc. which enabled adequate frequency response (better than 100 c.p.s.) to be obtained. Perhaps the biggest advantage was that calibration could be carried out by movement of a transmitter magslip in the recording room. The experimental arrangement is shown in Figure 35. Typical records are shown in Figure 33.

The rotor of the transmitter magslip is energised at 10V r.m.s. by a 1 Kc. oscillator. The rotor winding induces voltages into the delta connected stator windings which is passed on to the receiver magslip. Here the three stator windings give a resultant field so that the voltage appearing across the rotor varies according to its orientation, the output being fed to the oscilloscope.

The rocket is connected to a lever system which renders the magslip sensitive to horizontal bending but insensitive to vertical bending. An important feature is the use of piano wire at the right angle joint which avoids the uncertainty of backlash at a pin joint. This useful device was suggested by Mr. J. Waddington. Rotation of the transmitter rotor is equivalent to moving the rocket and hence calibration is carried out by movement of an arm connected to the rotor to known distances, usually 0.25" on either side of the zero line.

2.2 Discussion of the experimental results

In the following pages are given the results of the bending experiments using both coated and uncoated 3-inch rocket tubes. The numbers of rounds in the various categories fired are not sufficient to determine the effect of the coating. While it is apparent that bending in the first tenth of a second is less in the case of coated tubes, the bending thereafter in each case is too variable to permit quantitative comparison. The tests have been carried out with the various charges as follows:-

Tubular charges

Results of the measurement of the bending of 3-inch rockets filled with 2.7 - 0.75 inch tubular charges are shown in Figure 4. The most prominent cause of bending in this case is probably the offsetting of the charge from the tube axis. This leads to several effects:- Firstly, the rate of burning of cordite in the region of a steel wall is reduced and hence this wall is likely to be the cooler. Secondly, however, the conduit on the side nearest the wall is narrower and increased heat transfer should result. Our present knowledge of flow inside a rocket with an offset charge does not allow the heat transfer effect to be calculated, since the flow is essentially two dimensional. We have found, however, that rockets having charges offset one tenth of an inch from the tube axis, bend violently in the direction of the narrow conduit; the results are shown in Figure 5. This offset is, of course, exaggerated beyond that likely to be met in practice though not so greatly as appears at first sight. There is normally allowed a clearance of the order 0.050 inch at 70°F., to ensure easy filling and reasonable fitting at the upper temperature. In flight the charge is overloaded as a strut and becomes bowed between the walls giving rise to conditions tending to those obtained with the above offset charge during static firing.

The tabs used to position the charge are expected to be the cause of secondary effects. They are normally arranged in the form of a spiral, each tab causing a disturbed wake in the gas stream which is not diametrically balanced. The rate of burning in the region of the tabs is altered by erosion and variable stream pressure. Solvent from the cement used to attach the tabs to the cordite causes a reduction in burning rate and the cellulose acetate itself used for tabbing is attacked rapidly by the cordite gases and tends to cool the stream locally.

Opposite the tabs are small holes which prevent the unstable burning which is associated with secondary peaks. In the region slightly down stream of these holes, temperature sensitive paints show marked contours which indicate that the gases gush on to the tube at these points. Figure 6 shows temperature differences measured across a diameter 10.25 inches from the venturi end of the rocket.

Cruciform charges

In common with all charges which expose the tube directly to the main gas flow, bending arises from wall thickness variations and insulating coating of uneven thickness. In addition, and characteristic of the cruciform, is the possibility of one arm making poor contact with the tube, leading to a strip of the tube being raised to a higher temperature than its symmetrically placed counterpart. This effect is difficult to avoid because of the filling clearance needed. Further, the presence of different sized conduits must be considered as they may arise from offsetting of the charge or as a result of extrusion from an uneven die during manufacture. The heat transfer rate is dependent on the conduit area. The results of bending trials on Service rockets using the present standard X11 charge are given in Figure 7.

The temperature reached in X11 filled rounds was calculated in (Ref.1) Figure 8. The results of the present thermocouple measurements are given in Figures 9 and 10. Agreement with theory is satisfactory, except possibly near the head end of the rocket where the observed temperatures are somewhat higher than predicted. The positions of the thermocouples are shown in Figure 11.

Cruciform charge in motor tube with special insulating linings

Firings were carried out using tubes lined with 0.050 inch of an insulating rubber which was fitted into the tube so that the seam was a tightly sprung joint. Standard X11 charges were modified by planing 0.050 inch of cordite off each arm before applying the inhibiting strips to allow room in the motor tube for the lining. At the end of burning at least half the tube is still protected by the lining the remainder of the lining being consumed. The results of this trial shown in Figure 12 indicate that the bending is much reduced. The rapid bending of one round at the end of 0.6 seconds is considered due to the breakdown of insulation on one side.

An independent investigation of materials intended for thermal insulation linings (Ref. 5) showed that polythene was very effective and work has been commenced to line tubes evenly with this material. Heating of the tube is avoided for about one second by using a 0.068" thickness of polythene Figures 13 and 14. A second more readily available material which showed promise was rubber BY16B (Ref. 5) and some seamless linings were fitted to 3-inch rocket tubes. The material used was poorly fabricated and the wall thickness varied from 0.025 - 0.075 inch. On firing, the tubes remained relatively straight until burn through (0.5 sec.) and then bent violently, presumably due to the insulation failing earlier on the thin side, Figure 15. Temperature recordings taken simultaneously with the bending trials confirmed that bending commenced at burn through.

Efforts are now being concentrated upon methods of producing an even coating of polythene for the lining.

Star centre charges

If no device is used to seal one or other end of an inhibited charge in a rocket then, as a result of the pressure differential, gas flow occurs over the inhibiting coating on the outside of the charge. This is accompanied by heating of the tube, characteristically at the head end of the charge, the gases being cooled as they flow through the narrow conduit between charge and tube towards the venturi. This effect is most marked when the conduit area at the end of the charge approaches that of the venturi throat and the pressure drop is high. Under these conditions the result is occasionally disastrous as the inhibiting coating is eroded away.

Figure 16 shows the distribution of maximum temperatures in an experimental spin stabilised rocket of 100 mm calibre with open ended star charge. The temperature rise at the head end is clearly indicated. At the venturi end, heating arises by conduction back from the venturi plate, the temperatures indicated by the temperature sensitive paints occurring several seconds after the termination of burning. Similar results occur in the 3-inch rocket when filled with an open ended star (Figures 17, 18 and 19). It has been noticed that the heating varies considerably in magnitude from round to round, but the temperature pattern is retained.

In view of the difficulties mentioned above it seemed desirable to seal the charge either at the head or venturi end so that the differences in temperature likely to arise around the tube could be avoided or minimized. For static firing, it is easiest to seal the charge with a disc at the head end; although the possibility of fracture of the charge on ignition is admitted in so doing, no such result has yet occurred in firings at various temperatures.

A trial using SU/K star centre charges resulted in very high bending even though the head end of the charge was sealed (Figure 20). Similar results were obtained using an improved charge in the platonised propellant, PU (Figure 21). These results were surprising and tube temperatures were measured in an endeavour to establish the cause. It was at once apparent (Figure 22) that the temperatures reached were excessive, and could not be explained purely on the basis of gas flow into the annular space around the

charge followed by attainment of thermal equilibrium with the wall (Appendix 1). The heat content of the tube was some thirteen times too high to account for the temperature rise in this way and it appeared that excess gas flow must have been taking place towards the head end during burning. The head-end obturator used to seal the shell ring was suspected and this was confirmed by the fact that adhesive tape wrapped round the shell ring studs was set alight on firing.

The 3-inch rocket was accordingly modified so that the head end of the tube was sealed by a welded end plate. The bending of the rocket was greatly reduced as shown in Figure 23. It appears therefore that bending can be largely prevented provided adequate attention is given to the sealing of the head end of the rocket and the charge. The residual bending is readily explained by the fact that heating occurs from the gas which equalises the pressure in the conduit and the outer annulus between inhibitor and wall.

Temperature sensitive paints confirmed that temperatures were low, though conduction back from the venturi obscures the actual values that arise during burning. The temperatures indicated by Tempilac paints in the cases of ordinary obturators and welded seals are compared in Figure 24.

Later trials with thermocouples confirmed this view (Figure 25), the venturi end thermocouple indicated a high temperature with this assembly as the heat is conducted from the venturi block. Service rockets must be non-selfpropulsive when stored and the welded end disc would not be permissible. In consultation with E.R.D.E. the possibility of sealing the head end obturator with a cement was examined and a polymer made from cashew-nut oil and para-formaldehyde CS which polymerized at air temperature was tried. Figure 26 shows the results of a bending using the above mentioned cement to seal the head-end obturator. The bending is of low order and similar to that obtained with welded head seals, but with loose charges the bending is appreciable.

Records are available of star-centre charges used in the 3-inch rocket during the early part of the late war by the Low Pressure Ballistics Section at Woolwich.(Ref. 8). At the time, the technique of inhibition was in an early stage of development and layers of tape, bonded with P.Q. cement were used. To prevent the gas from ripping the inhibiting tape away, a reversed obturator was used at the head end and this appears to have functioned as a head end seal. Projections of these charges showed that they had a relatively low dispersion; particularly so, because the thrust was lower than that of tubular filled rockets and of longer duration, giving a lower launch velocity and raising the expected dispersion. Some of the improvement was ascribed to the venturis which were machined from the solid and which gave a slightly improved dispersion with tubular charges. Novertheless, several trials were carried out showing that star centre charges were associated with a low dispersion in this particular rocket.

Preliminary work with rockets filled with plastic propellant is described in (Ref. 9). In this case emphasis is placed on the centralizing of the central conduit in order to avoid extremely large end deflection of the order 1-2 inches which occur due to uneven sliver left at burn out. Lift forces are then liable to deviate the rocket since the velocity is high. No bending occurs with case bonded plastic propellant filled rockets until the sliver period since the tube is completely insulated.

3. DERIVATION OF THE EQUATIONS OF MOTION OF THE NON-RIGID ROCKET IN FLIGHT

The motion of the rigid rocket in flight has been described in detail by many writers. The deviations of the rocket from the flight trajectory due to such factors as tolerances and wind have been calculated for many weapons. An account of the work and the methods of calculation are given by Davies (Ref. 7). In the calculations given there it has been assumed that the rocket behaves as a rigid body i.e. it does not bend. In this section we extend the theory to the non-rigid rocket, i.e. a rocket which bends uuder the stresses arising and the temperature effects. The derivation of the equations of motion

is described in detail so that the reader may comprehend the extent of the analysis and the assumptions made. The method of solution of these equations is complex and exceedingly lengthy and is not given here; the conclusions from the analysis only are given. A detailed account of the method of solution will be given in a later report.

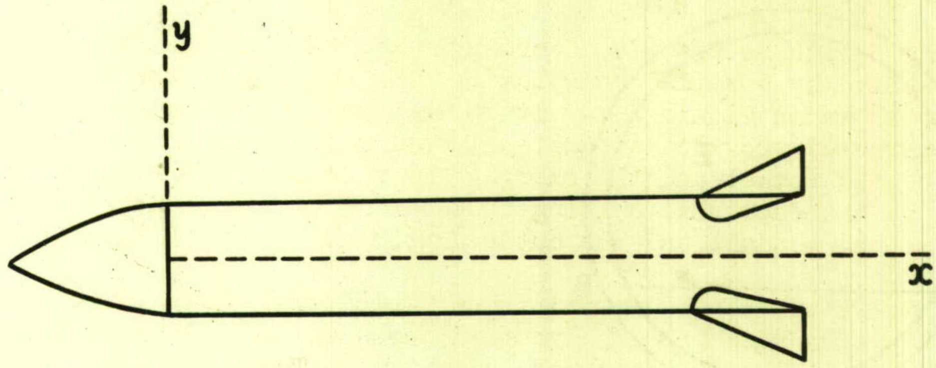


Fig. 3.1

The x axis is taken along the central axis of the rocket and $x = o$ is taken at the line of intersection of the tube and shell assembly. The line of intersection of the nozzle assembly and the tube we take as $x = l$. The y axis is taken at right angles to the x axis and the (x,y) plane is the plane of motion. We adopt the convention that the shearing force S at any section x is the sum of the external forces in the vertical direction y on the right hand side of x while the bending moment B is the sum of the anti-clockwise moments of these forces about x. The wall thickness of the tube is t and the inside and outside radii are r_1 and r_2 respectively. The shell assembly of mass W_1 and the nozzle assembly of mass W_2 we treat as rigid bodies.

The coordinates (x,y) specify the position of an elementary length of the rocket axis. In the first instance let us suppose that there are no external forces on the system (except, of course, d'Alembert forces). We suppose the rocket executes small motions about its initial direction. The terms $\frac{\partial y}{\partial x}$, $\frac{\partial^2 y}{\partial x^2}$ are all taken as small and we neglect their squares and products. Inside the tube is the rocket charge which rests on the grid at the nozzle end. It is not anchored at the head end but plastic tabs placed in contact with the tube and charge restrict their independent motion. In this report we suppose the tube and charge bend and vibrate in phase with equal amplitude. The effect of this assumption is considered in a later supplementory report.

The stress systems arising in the rocket tube are

(i) the elastic forces due to the bending of the rocket,

(ii) the reactions of the gas flowing in the conduit,

(iii) the inertia forces,

(iv) thermal stresses.

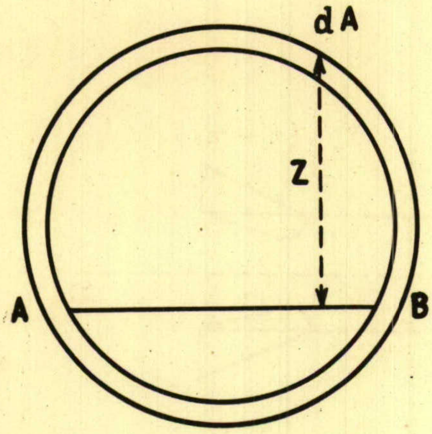


Fig. 3a

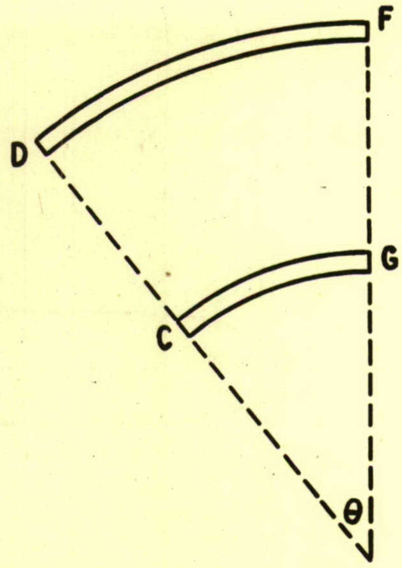


Fig. 3b

The section of the tube in Figure 3a is perpendicular to the x axis, and AB is an arbitrary plane. The perpendicular distances from this plane are denoted by z. The elementary length of the rocket tube of which Figure 3a is the section is given in Figure 3b. In its initial state this section has a radius R_0 and subtends an angle θ_0 at the centre of the circle of radius R. In its strained state let these values be R and θ respectively. We suppose the temperatures of the elements of the tube are functions of z so that we consider the two dimensional problem only. The temperature of the tube in the initial state is taken as the zero of temperature. If k_z is the mean coefficient of thermal expansion between the zero and T_z, the extension of the longitudinal element at z, which gives rise to stress, is

$$\left[(R + z)\theta - (k_z T_z + 1)(R_0 + z)\theta_0\right]/(R_0 + z)\theta_0 \ , \qquad (3.1)$$

T_z being the temperature at z.

If at a plane z = a, the extension is zero

$$(R + a)\theta = (R_0 + a)\theta_0(1 + k_a T_a) \qquad (3.2)$$

In terms of this, the extension can be written

$$(z - a)\left(\frac{1}{R} - \frac{1}{R_0}\right) + (k_a T_a - k_z T_z) \ , \qquad (3.3)$$

neglecting the squares and products of $1/R$ and $1/R_0$. If E_z is longitudinal Young's modulus at z, the force on the element is thus

$$E_z \left\{(z - a)\left(\frac{1}{R} - \frac{1}{R_0}\right) + k_a T_a - k_z T_z\right\} dA \ ; \qquad (3.4)$$

dA being the cross sectional area of the element. The total force across the section is

$$F_x = \oint E_z \left[(z - a)\left(\frac{1}{R} - \frac{1}{R_0}\right) + k_a T_a - k_z T_z \right] dA \,, \tag{3.5}$$

and the moment of the forces about $z = z_1$ is

$$\oint E_z \left[(z - a)\left(\frac{1}{R} - \frac{1}{R_0}\right) + k_a T_a - k_z T_z \right] (z - z_1)\, dA,$$

which may be written

$$- F_x z_1 + \mu\left(\frac{1}{R} - \frac{1}{R_0}\right) + T', \tag{3.6}$$

where

$$\mu = \oint (z - a) E_z z dA \,, \quad T' = \oint E_z z \,(k_a T_a - k_z T_z)\, dA, \tag{3.7}$$

the integrals being taken around the complete cross-section of the tube.

The moment of the forces about $z = z_1$, is equivalent to a force F_x parallel to the axis at $z = o$ and a couple of magnitude

$$\mu \left(\frac{1}{R} - \frac{1}{R_0}\right) + T' \tag{3.8}$$

The reaction on the tube of the gas in the conduit can be determined in the following manner. Let us assume the area of the gas conduit along the length of the motor is constant; departures from this assumption can be shown to be of little importance.

Let p be the pressure at CD in Figure 3a, the pressures at GF is $p + \frac{\partial p}{\partial x} dx$. In an interval δt, the momentum crossing CD is $\rho v^2 A \delta t$, ρ being the density and v the velocity of the gas stream and A the area of the gas conduit. In the interval δt the momentum flowing across FG is

$$A\left(\rho v^2 + \frac{\partial \rho v^2}{\partial x} \delta x\right) \delta t$$

Now suppose the section of the motor is at rest and also the gas is liberated from the charge with no momentum. If ΔS_p is the shearing force exerted on the motor by the gas stream, from the conservation of momentum in the vertical plane we have

$$\left[p \frac{\partial y}{\partial x} A - p \frac{\partial y}{\partial x} A - \frac{\partial}{\partial x}\left(p \frac{\partial y}{\partial x}\right) A - \Delta S_p \right] \delta t$$

$$= \left[\rho v^2 \frac{\partial y}{\partial x} + \frac{\partial}{\partial x}\left(\rho v^2 \frac{\partial y}{\partial x}\right) - \rho v^2 \frac{\partial y}{\partial x} \right] A \delta t \,,$$

or

$$\frac{\partial S_p}{\partial x} = - A \frac{\partial}{\partial x}\left[(p + \rho v^2) \frac{\partial y}{\partial x} \right]$$

From the conservation of momentum in the horizontal direction we have

$$p + \rho v^2 = P_0 \ ,$$

P_0 being the head end pressure, and thus

$$\frac{\partial S_p}{\partial x} = - P_0 \ A \frac{\partial^2 y}{\partial x^2} \ .$$

In this we have assumed the section of the motor to be at rest. If we include the motion in the vertical plane we have to add to the above the shearing force due to the inertia acceleration in the vertical plane and also the component due to the change in the direction of the gas stream. It is easily seen that

$$\frac{\partial S_p}{\partial x} = - P_0 \ A \frac{\partial^2 y}{\partial x^2} - 2A\rho v \frac{\partial}{\partial t} \left(\frac{\partial y}{\partial x}\right) - A\rho \frac{\partial^2 y}{\partial t^2} \ . \tag{3.9}$$

In these we have neglected the effect of the forward velocity of the rocket. This will be discussed in a later report.

If w is the weight of the motor tube per unit length, the longitudinal force at the section x is

$$F_x = P_0 A - W_1 f - wxf$$

In deriving the elastic stresses we used an arbitrary reference plane for $z = o$. We have also used a reference axis for y. Let the $z = o$ plane be specified by Y when referred to the $y = o$ axis. The ordinate y is on the resultant momentum vector of the gas flow in the conduit.

Consider the forces and couples in the elementary length of the rocket given in Figure 3b. On the section CD we have a vertical force $-S$ a clockwise couple B, a horizontal force $-F_x$ at $z = o$. On FG we have a vertical force $S + \frac{\partial S}{\partial x} dx$, an anti-clockwise couple $B + \frac{\partial B}{\partial x} dx$ at $z = o$. On the element there is an equivalent vertical force of

$$\frac{\partial S_p}{\partial x} dx - \frac{\partial}{\partial t} \left(w \frac{\partial y}{\partial t}\right) dx \ .$$

Equating vertical forces we find

$$-P_0 A \frac{\partial^2 y}{\partial x^2} - 2A\rho v \frac{\partial}{\partial t} \left(\frac{\partial y}{\partial x}\right) - A\rho \frac{\partial^2 y}{\partial t^2} - \frac{\partial}{\partial t} \left(w \frac{\partial y}{\partial t}\right) + \frac{\partial S}{\partial x} = o \ . \tag{3.10}$$

Taking moments we find

$$S + \frac{\partial B}{\partial x} - F_x \frac{\partial Y}{\partial x} = I_3 \frac{\partial^3 y}{\partial x \partial t^2} + wfe \tag{3.11}$$

where $I_3 dx$ is the moment of inertia of the element about an axis through its centre of gravity and perpendicular to the rocket axis and B is given by (equation 3.8).

$$B = - \mu\left(\frac{1}{R} - \frac{1}{R_0}\right) - T', \tag{3.12}$$

with

$$\frac{1}{R} = -\frac{\partial^2 y}{\partial x^2}$$

The term e is the distance of the mechanical centroid of the section above $z = o$. Differentiating equation 3.11 with respect to x we obtain $\frac{\partial S}{\partial x}$ which can be substituted into equation 3.10 to obtain the differential equation of motion.

In the preceding analysis two planes of reference have been employed namely $y = o$ and $z = o$, the ordinates of the latter when referred to $y = o$ being denoted by Y. Only the differentials of y and Y occur. The y plane is such that at the point x the resultant momentum vector of the gas stream in the conduit passes through y. The choice of Y is still arbitrary and this is chosen from later considerations. We have assumed in the analysis that the momentum vector of the gas stream is determined entirely by the elementary section of the motor so that y expressed the position of the elementary section of the motor tube.

We have now to derive the boundary conditions. Let us first consider the contribution to the shearing force and the bending moment from the shell and nozzle assemblies. W_1 is the mass of the shell assembly and I_1 its moment of inertia about an axis through the centre of gravity of the shell assembly and perpendicular to the central axis. W_2 is the mass of the nozzle and I_2 its moment of inertia about an axis through the centre of gravity of the nozzle assembly and perpendicular to the central axis. Let l_1 be the distance of the centre of gravity of the shell assembly from $x = o$ and l_2 the distance of the centre of gravity of the nozzle assembly from $x = l$. The velocity of the centre of gravity of the nozzle is

$$\left(\frac{\partial y}{\partial t}\right)_{x = l} + l_2 \left(\frac{\partial^2 y}{\partial x \partial t}\right)_{x = l}$$

Thus the contribution to the shearing force at $x = l$ of the inertia of the nozzle is

$$S_2 = - W_2 \left[\frac{\partial^2 y}{\partial t^2} + l_2 \frac{\partial^3 y}{\partial x \partial t^2}\right] \quad , \quad x = l \tag{3.13}$$

while the contribution to the bending moment is

$$B_2 = - W_2 l_2 \left[\frac{\partial^2 y}{\partial t^2} + l_2 \frac{\partial^3 y}{\partial x \partial t^2}\right] - I_2 \frac{\partial^3 y}{\partial x \partial t^2} \; , \quad x = l \; . \tag{3.14}$$

Similarly we find at $x = o$, the contributions to the shearing force and bending moment of the shell inertia are

$$S_1 = W_1 \left[\frac{\partial^2 y}{\partial t^2} - l_1 \frac{\partial^3 y}{\partial x \partial t^2}\right] \quad , \quad x = o, \tag{3.15}$$

$$B_1 = - W_1 l_1 \left[\frac{\partial^2 y}{\partial t^2} - l_1 \frac{\partial^3 y}{\partial x \partial t^2}\right] + I_1 \frac{\partial^3 y}{\partial x \partial t^2} \; , \quad x = o. \tag{3.16}$$

Let the resultant force on the nozzle, the grid and the base of the charge be Q in the direction of x increasing. The total longitudinal force on the charge and tube at $x = l$ is

$$F_l = P_0 A - W_1 f - wlf = Q + W_2 f.$$

This is the equation for f the acceleration. Owing to the curvature of the tube we have an additional shearing force at $x = l$ of $\left(Q \frac{\partial y}{\partial x}\right)_{x = l}$; this we add to S_2. Similarly we add a shearing force of $P_0 A\left(\frac{\partial y}{\partial x}\right)_{x = o}$ to S_1.

Let us suppose the centre of pressure at the head end is at a distance e_2 above $z = o$, thus we add to B_1

$$- W_1 f\ l_1 \left(\frac{\partial y}{\partial x}\right)_{x = o} - P_0 A e_2 + W_1 f(e_3 + e_2)$$

where e_3 is the distance of the C. of G. of the shell assembly above the centre of pressure in the static undeformed state. This is made more definite when the axes have been defined. Similarly to B_2 we add

$$- W_2 f l_2 \left(\frac{\partial y}{\partial x}\right)_{x = l} - Qe_4 - W_2 f\ (e_5 + e_4)\ ,$$

where e_4 is the distance of $\vec{Q}$ above the plane $z = o$ and e_5 the distance of the centre of gravity of the nozzle assembly above $\vec{Q}$. The exact specification of the terms is given later when the axes are defined.

Further if due to the asymmetry of the nozzle, the line of thrust is inclined at an angle ψ and displaced laterally upwards above $z = o$ by an amount e_6 we add $- F\psi$ to S_2 and Fe_6 to B_2

Finally the gas flow in the nozzle gives a shearing force at $x = l$ of

$$- 2A\rho v l_3 \frac{\partial^2 y}{\partial x \partial t}$$

and a bending moment of

$$- A\rho v l_3^2 \frac{\partial^2 y}{\partial x \partial t}\ ,$$

l_3 being the length of the nozzle. These expressions we add to S_2 and B_2 respectively.

In addition to these we have external forces and couple. We denote these by a lateral body force of $J(x,t)$ per unit length and lateral forces and couples of $K_1(t)$, $H_1(t)$ amd $K_2(t)$, $H_2(t)$ at $x = o$ and $x = l$ respectively.

The resulting values of S_1, B_1 and S_2, B_2 we put equal to the values of S and B given by equations 3.13 onwards to obtain our boundary conditions.

To illustrate the form of the solution the differential equation and boundary conditions are derived for a perfect rocket. The complete specification and solution of the differential equation and boundary conditions are given in a later report.

The following assumptions were made.

(i) The mechanical centroid of any section coincides with the centre of pressure in the conduit,

(ii) The Young's modulus is constant around the section.

In this case we take the reference axix $z = o$ as the mechanical centroid of the section so that the axis of y and Y coincide. Also we have

$$\oint z\, E_z\, dA = 0, \quad \mu = E \oint z^2\, dA$$

and

$$T = - E \oint k_z\, T_z\, z\, dA ,$$

thus if we assume the conductivity and the temperature to be completely symmetrical about the centroid of the section we have $T = 0$. Also we omit the "gas whip" terms in $A\rho v$.

With these assumptions the differential equations and boundary conditions become

$$\mu D^4 y + (W_1 + wx) f D^2 y + wf Dy - I_3 \frac{\partial^2}{\partial t^2} D^2 y + w \frac{\partial^2 y}{\partial t^2} = J_1 (x, t), \quad o < x < l \qquad (3.17)$$

$$- \mu D^3 y - W_1 f Dy + I_3 \frac{\partial^2}{\partial t^2} Dy = W_1 \left[\frac{\partial^2 y}{\partial t^2} - l_1 \frac{\partial^2}{\partial t^2} Dy \right] - K_1(t), \quad x = o, \qquad (3.18)$$

$$- \mu D^3 y + W_2 f Dy + I_3 \frac{\partial^2}{\partial t^2} Dy = - W_2 \left[\frac{\partial^2 y}{\partial t^2} + l_2 \frac{\partial^2}{\partial t^2} Dy \right] + K_2(t), \quad x = l, \qquad (3.19)$$

$$\mu D^2 y = - W_2 l_2 \left[\frac{\partial^2 y}{\partial t^2} + l_2 \frac{\partial^2}{\partial t^2} Dy \right] - I_2 \frac{\partial^2}{\partial t^2} Dy - W_2 f l_2\, Dy + H_2(t), \quad x = l, \qquad (3.20)$$

$$\mu D^2 y = - W_1 l_1 \left[\frac{\partial^2 y}{\partial t^2} - l_1 \frac{\partial^2}{\partial t^2} Dy \right] + I_1 \frac{\partial^2}{\partial t^2} Dy - W_1 f l_1\, Dy - H_1(t), \quad x = 0. \qquad (3.21)$$

The equations of motion have been solved for the Rocket Aircraft 3-inch No. 1 rocket with parallel sided shell and A.D. fuze fired from a two railed launcher 12 ft. in length. In the numerical calculations the value of Young's modulus has been taken to be that for a cold tube so that the heating of the tube has been neglected. The conclusions from the analysis are as follows:-

(i) The fundamental frequency of the motor tube is 87 cycles per second.

(ii) As a result of this high fundamental frequency the deviation of the rocket due to small lateral forces is almost identical to that of a rigid rocket.

(iii) The bending due to the high internal stresses and the tolerances produce deviations which are small compared with the deviations in flight; the maximum mean deviation calculated being 0.03 degs., this being due to the known wall thickness variations.

(iv) The velocity at which the tube "flutters" due to the aerodynamic forces is considerably in excess of the velocities attained by this rocket. (At the flutter velocity resonance vibration takes place).

In the calculations it has been assumed that the fins are rigid, no account has been taken of fin vibration.

Finally we conclude from our analysis that if the heat is kept from the tube, the assumption of non-rigidity made in the classical rocket theory is valid. In the next section, however, it is shown that the temperature effects are serious.

Section 4. The bending of the rocket due to temperature effects

In the previous section we have, in the numerical analysis, assumed that the temperature is constant around a section of the motor and also that Young's modulus is constant. In view of the high temperatures attained by the tube we now investigate temperature effects.

We have seen that with a perfect rocket i.e. uniform wall thickness and symmetrical temperature distribution the pressure has no effect on the bending. Further the set-back forces create an effective compression on the tube but with the 3-inch rocket already investigated they have little effect on the motion.

In the first instance we assume the rocket to have a uniform wall thickness but that the tube has a temperature variation around the section, there being no variation along the length. We suppose the temperature variation sufficiently small to have a negligible effect on the Young's modulus.

As before we take the z = o axis such that $\oint z\, E_z\, dA = 0$ so that

$$\mu = E \oint z^2\, dA\ , \qquad T' = -\,k\, E \oint Tz dA\ .$$

The bending moment at the section is

$$B = \mu/R + T'\ ,$$

and if we neglect the effect of the set back forces the mean bending couple B is zero so that

$$1/R = -\,T'/\mu\ .$$

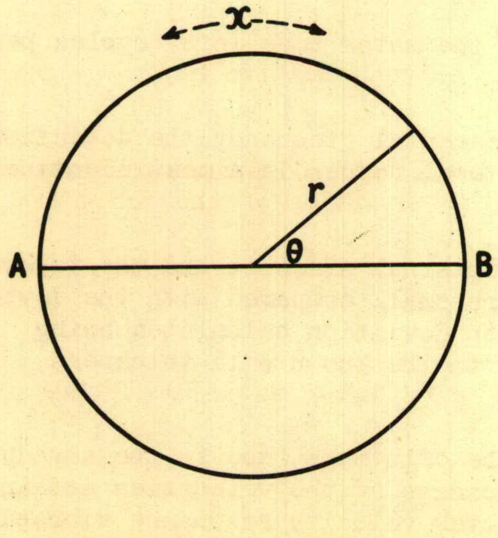


Fig. 4.1

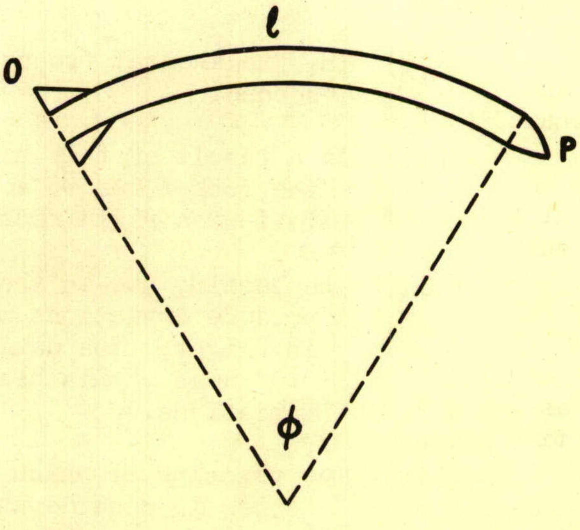


Fig. 4.2

Assuming the tube to be circular, r the mean radius and a the wall thickness we have

$$1/R = k \int_0^{2\pi} T\, r^2 \sin\theta\, a\, d\theta \Bigg/ \int_0^{2\pi} a\, r^3 \sin^2\theta\, d\theta ,$$

$$1/R = \frac{k}{\pi r} \int_0^{2\pi} T \sin\theta\, d\theta .$$

Let us suppose the upper half of the tube is T degrees above the lower half i.e. the temperature above A B is T degrees higher than that below. Hence

$$1/R = \frac{kT}{\pi r} \int_0^{\pi} \sin\theta\, d\theta = 2\, k\, T/\pi\, r .$$

The angle ϕ subtended by the rocket motor tube is

$$\phi = l/R = 2\, k\, l\, T/\pi\, r .$$

For the 3-inch rocket with parallel sided shell and A D fuze it can be shown that if the angle subtended by the tube is ϕ, the centre of gravity is at a perpendicular distance from the line of thrust which to a very good approximation is $l\, \phi/2$. Thus the effective nozzle offset is $\phi/2$, where

$$\phi/2 = k\, l\, T/\pi\, r \tag{4.1}$$

Taking $k = 10^{-5}$, $l = 3.6$ ft., $r = 1.59$ ins. we find

$$\phi/2 = 8.64\, T \times 10^{-5}.$$

An effective nozzle offset of 10^{-3} radians causes a deviation in flight of 0.8 degs. Thus the deviation in flight is α where

$$\alpha = 8.64 \times 0.8 \times T \times 10^{-2},$$

and thus a temperature difference of 14.5°C between the two halves of the tube causes a deviation in flight of one degree.

If a strip of width x along the whole length of the rocket tube were at a temperature T above or below that of the remainder of the tube, the subtended angle is

$$\phi = l\, k\, T\, x/\pi\, r^2$$

assuming x is sufficiently small for $\sin\theta$ to be taken as unity. The effective nozzle offset is

$$\phi/2 = k\, l\, T\, x/2\, \pi\, r^2$$

and the deviation in flight is

$$\alpha = 0.8 \times 10^{-2}\ l\ T\ x/2\pi r^2 \tag{4.2}$$

Taking $x = 1$ inch, a temperature difference of 46°C causes a deviation in flight of one degree.

These temperatures appear small when compared with the high temperatures attained in the tubes of the 3-inch motors but it must be noted that we have stipulated these temperature differences as applying along the whole length of the tube, an unlikely condition in practice.

Measurements of the wall thicknesses of a large number of tubes have shown that the wall thickness varies systematically around the section and this variation is maintained with only slight change along the whole length of the tube. The results of the measurements are given in appendix I. If with tubes of this type we chose the reference plane $z = o$ to coincide with the line of centroids, the centre of pressure in the conduit no longer coincides with $z = o$. The rocket is bent by two factors

(i) the temperature variation resulting from the uneven wall thickness,

(ii) due to the separation of the centre of pressure from $z = o$, the gas pressure exerts a bending couple on the tube.

With uncoated tubes and tubes coated with 0.006 ins. of refractory coating, the former is markedly greater than the latter and for the moment we concern ourselves with the effect of the temperature variation.

We choose our reference plane for z such that

$$\oint z\, E_z\, dA = 0\ , \tag{4.3}$$

and this defines our reference plane uniquely. Thus we find

$$\mu = \oint z^2\, E_z\, dA\ ,\quad T' = -\oint E_z\, z\, k_z\, T_z\, dA\ . \tag{4.4}$$

Let us suppose the heat transmission coefficient from the gas to the metal is constant all around the section. During the time of burning there is negligible heat flow around the tube, the heat flow being almost entirely radial and the local mean temperature of the tube is dependent on the wall thickness at the point. For example in Figure 2a, the mean temperature of the element dA is dependent on the wall thickness at that point.

The temperature depends on the depth u from the inside surface. The value of z is practically constant over the element and by the temperature T_z we imply the mean temperature over dA or

$$T_z = \frac{1}{L}\int_0^L T\, d u\ . \tag{4.5}$$

L being the local wall thickness.

Let $\bar{L}$ be the mean wall thickness over the cross-section of the tube and let $\theta_m(\bar{L})$ be the mean wall temperature with this value $\bar{L}$. Thus for small variations of L about $\bar{L}$

$$\int_0^L T\,d\,u \equiv T_z\,L = L\left\{\theta_m\,(\bar{L}) + \frac{\partial\theta_m}{\partial L}\,(L - \bar{L})\right\}. \tag{4.6}$$

It is shown from the measurements of wall thicknesses of a large number of tubes given in the appendix, that the wall thickness at z is given by a relation of the form

$$L = \bar{L} + b\,(z - z_2)/r \tag{4.7}$$

r being the radius of the tube.

Thus we have

$$\oint z\,T_z\,dA = \theta_m\,(\bar{L})\oint z\,d\,A + \frac{\partial\theta_m}{\partial L}\left[\frac{b}{r}\oint (z - z_2)\,zdA\right],$$

$$= \left[\theta_m\,(\bar{L}) - \frac{bz_2}{r}\,(\frac{\partial\theta_m}{\partial L})\right]\oint z\,d\,A + \frac{\partial\theta_m}{\partial L}\,\frac{b}{r}\oint z^2 dA\,. \tag{4.8}$$

The Young's modulus E_z is approximately a linear function of temperature and we write

$$E_z = E_0 + E_1\,T_z\,, \tag{4.9}$$

and thus

$$\oint E_z\,z\,d\,A = E_0\oint z\,d\,A + E_1\oint z\,T_z\,d\,A$$

or

$$\oint E_z\,z\,d\,A = \left\{E_0 + E_1\left[\theta_m\,(\bar{L}) - \frac{b}{r}\,z_2\,(\frac{\partial\theta_m}{\partial L})\right]\right\}\oint z\,d\,A$$

$$+ E_1\,(\frac{\partial\theta_m}{\partial L})\,\frac{b}{r}\oint z^2\,d\,A\,, \tag{4.10}$$

and thus since the left hand side has been equated to zero to specify the axis we have

$$\oint z\,d\,A = -\frac{E_1\,\dfrac{\partial\theta_m}{\partial L}\,\dfrac{b}{r}\oint z^2\,d\,A}{E_0 + E_1\left[\theta_m\,(\bar{L}) - \dfrac{b}{r}\,z_2\,(\dfrac{\partial\theta_m}{\partial L})\right]}\,. \tag{4.11}$$

The values of E/E_0, the ratio of Young's modulus at the specified temperature to that at zero, together with the values of k and k E/E_0 for medium carbon steel are given in the following table.

Temp. ($^{\circ}$C.)	E/E_0	$k \times 10^5$	$kE/E_0 \times 10^5$
0	1.0	1.15	1.15
50	0.985	1.17	1.18
100	0.970	1.20	1.21
150	0.955	1.23	1.24
200	0.94	1.25	1.27
250	0.92	1.27	1.29
300	0.90	1.30	1.30
350	0.88	1.33	1.32
400	0.86	1.35	1.33
450	0.83	1.37	1.33
500	0.81	1.40	1.32
550	0.71	1.43	1.21
600	0.60	1.45	1.05

The product kE varies with temperature and the expression for T given by equation 4.4 becomes complicated. It is considerably simplified if we suppose $k_z E_z$ is a function T_z as given by the above table. Let us write

$$E_z k_z = (Ek)_m + \frac{\partial Ek}{\partial T} (T_z - \theta_m) , \qquad (4.12)$$

$(Ek)_m$ being the value at θ_m. The moment of area I of the section about z = o is

$$I = \oint z^2 dA ,$$

and in equation 4.11 the term $bz_2 \frac{\partial \theta_m}{\partial L}$ can be neglected compared with θ_m and thus

$$\oint z \, d A = - E_1 \, b \, I \, \frac{\partial \theta_m}{\partial L} \Big/ E_m r , \qquad (4.13)$$

E_m being the Young's modulus at the temperature θ_m $(\bar{L})$.

In appendix 2, it is shown that for the 3-inch rocket tube, wall thickness 0.08 ins.

$$\left(\frac{\partial \theta_m}{\partial L}\right)_{L=\bar{L}} = - 11.2 \, (\theta_1 + \theta_2)/2 ,$$

θ_1 and θ_2 being the inner at outer temperature of the tube wall. Thus

$$\oint z \, d A = 5.6 \, (\theta_1 + \theta_2) E_1 \, b \, I/E_m r , \qquad (4.14)$$

and thus from equations (4.8, (4.12) and (4.13)

$$T = - (Ek)_m \left[\theta_m \, (\bar{L}) \left\{ 5.6 \, E_1 bI \, (\theta_1 + \theta_2)/E_m r \right\} - 5.6 \, (\theta_1 + \theta_2) bI/r \right]$$

$$- \frac{\partial Ek}{\partial T} \oint T_z z\, (T_z - \theta_m)\, d\,A\ ,$$

$$= 5.6\, (\theta_1 + \theta_2) \left[(Ek)_m\, bI\, (E_m - E_1 \theta_m)/E_m r + \frac{\partial Ek}{\partial T} \oint T_z (L - \overline{L})\, zdA \right] .$$

The integral expression may be written

$$\oint T_z\, (L - \overline{L})\, zdA = \frac{b}{r} \oint T_z z^2 dA\ ,$$

and to a sufficient approximation the term T_z in this way may be replaced by θ_m and thus

$$T' = 5.6\, (\theta_1 + \theta_2) \frac{bI}{r} \left[(Ek)_m\, E_0/E_m + \frac{\partial Ek}{\partial T}\, \theta_m \right] .$$

We have

$$\mu = \oint z^2 E_z\, dA\ ,$$

and to a sufficient approximation this may be written

$$\mu = E_m I\ ,$$

and thus

$$1/R = - \frac{T'}{\mu} = 5.6\, (\theta_1 + \theta_2) \left[(Ek)_m\, E_0/E_m + \frac{\partial Ek}{\partial T}\, \theta_m \right] /E_m\ , \qquad (4.15)$$

and this may be written (since $k_m = k_0 + \frac{dk}{dT}\, \theta_m$)

$$1/R = 5.6\, (\theta_1 + \theta_2) \frac{b}{r} \left[k_m + \frac{E_0}{E_m}\, \frac{dk}{dT}\, \theta_m \right] . \qquad (4.16)$$

It remains to calculate the position of the reference axis z = o. To simplify the solution we suppose the internal contour of the cylinder is a perfect circle of radius r. Thus

$$\oint z\, d\, A = \int_0^{2\pi} (r \sin\theta - a)\, rd\theta \left[\overline{L} + \frac{b}{r}\, (z + a) \right] ,$$

where a is the distance of the reference axis z = o above the diameter of the circle. Thus

$$\oint z\, d\, A = b(\pi r^2 + 2\pi a^2) - 2\pi ra\, (\overline{L} + \frac{b}{r}\, a)\ .$$

Equating this to the value given by equation (4.14) we find (neglecting a^2 compared with r^2)

$$a = \frac{br}{2} \frac{(1 - K\bar{L})}{\bar{L} - Kb^2} , \qquad (4.17)$$

where $K = 5.6 (\theta_1 + \theta_2) E_1/E_m$.

The temperatures in 2-inch and 3-inch motors have been determined theoretically (Ref. 1) and the temperatures on the outside of the tube have been determined experimentally at A.R.D.E. and P.D.E. (Ref. 2). The measurements and calculations have been confined to three positions along the tube namely (i) near the head end (ii) near the nozzle end and (iii) at the central section. At the nozzle end and at the middle sections the theory and experiment agree reasonably well but at the head end the observed outside temperatures are appreciably greater than those calculated. The theoretical values of the temperature at the inside wall are readjusted using a heat transmission coefficient which gives the observed outside temperature. With sufficient accuracy θ_m may be replaced by $(\theta_1 + \theta_2)/2$. The radii of curvature have been calculated for the 3-inch rocket with the tubular charge at 20°C and with $b = 10^{-3}$ ins. The curvatures are given in the following table, R being in inches.

	Nozzle End		Head End		Middle Section	
t (secs.)	$\theta_1 + \theta_2$	$1/R \times 10^5$	$\theta_1 + \theta_2$	$1/R \times 10^5$	$\theta_1 + \theta_2$	$1/R \times 10^5$
0.05	276	1.31	112	0.49	194	0.88
0.15	608	3.46	292	1.40	488	2.48
0.25	858	5.50	406	2.08	658	3.68
0.35	982	6.66	482	2.56	792	4.76
0.45	1108	8.20	554	3.01	900	5.70
0.55	1200	9.76	610	3.35	988	6.56
0.65	1284	12.20	672	3.75	1068	7.51
0.75	1350	*	700	3.93	1148	8.23
0.85	1398	*	732	4.15	1228	10.14
0.95	1440	*	762	4.34	1280	11.87

*E_0/E_m is not known accurately at these temperatures - it increases very rapidly giving rise to very high bending.

At a specified time the curvature varies along the length. We assume it can be expressed in the form of a quadratic, or

$$\frac{d^2y}{dx} \equiv 1/R = a_1 + b_1 x + c_1 x^2$$

Writing $1/R_1$, $1/R_2$ and $1/R_3$ as the curvature at the head end, the middle section and the nozzle end respectively, we can express a_1, b_1 and c_1 in terms of these. The angle between the nozzle axis and the rocket axis at the head is thus

$$\left[\frac{dy}{dx}\right]_{x=0}^{l} = a_1 l + b_1 l^2/2 + c_1 l^3/3 = (1/R_1 + 4/R_2 + 1/R_3) l/6 ,$$

l being the length of the motor. This angle is calculated at the various times

t (secs.)	Angle between shell axis and nozzle axis due to bending
0.05	0.41×10^{-3} radians
0.15	1.11 " "
0.25	1.70 " "
0.35	2.12 " "
0.45	2.55 " "
0.55	2.95 " "
0.65	3.45 " "
0.75	3.70 " " *
0.85	4.56 " " *
0.95	5.34 " " *

*These are taken to be $1/R_2$, this underestimates the angle.

In the bending experiments referred to in other sections the head end of the rocket is rigidly clamped and the lateral movement of the venturi end measured. This lateral movement is

$$[y]_0^l = a_1\, l^2/2 + b_1\, l^3/6 + c_1\, l^4/12 = l^2\, [1/6R_1 + 1/3R_2]\ , \qquad (3.18)$$

The calculated movements for the 3-inch rocket with tubular charge and $b = 10^{-3}$ ins. are given in the following table.

t (secs.)	Lateral movement of nozzle end with head end clamped (ins.)
0.05	0.007
0.15	0.021
0.25	0.032
0.35	0.041
0.45	0.049
0.55	0.056
0.65	0.063
0.75	0.069
0.85	0.082
0.95	0.095

Due to the curvature of the tube the centre of gravity moves laterally. If $\overline{y}$ is the lateral movement of the C. of G., d_1 the lateral movement of the end $x = l$ and ϕ the angle between the shell axis and the nozzle axis, the effective tolerance angle is

$$\alpha = \phi - (d_1 - \overline{y})/l - e)\ ,$$

e being the distance of the C. of G. from $x = o$. The term $\overline{y}$ is given by the equation

$$(W_1 + wl + W_2 l_2)\, \overline{y} = w\, [a_1 l^3/6 + b_1 l^4/24 + c_1 l^5/60]$$

$$+ W_2\, [(a_1 l + b_1 l^2/2 + c_1 l^3/3) l_2 + (a_1 l^2/2 + b_1 l^3/6 + c_1 l^4/12)]$$

the notation being as in section 3. The calculated values are given in the following table.

t	Equivalent nozzle offset (radians x 10^{-3})
0.05	0.28
0.15	0.71
0.25	1.09
0.35	1.32
0.45	1.61
0.55	1.88
0.65	2.17
0.75	2.46
0.85	3.00
0.95	3.91

Approximately the effective tolerance angle is a linear function of the time. Further the velocity of the rocket is approximately a linear function of the time. We have

$$\alpha = \beta + \gamma t \ , \ V = V_0 + ft \ , \ f = F/W \ .$$

V_0 being the launch velocity.

$$\alpha = \beta + \gamma \ (V - V_0)/f \equiv \beta_1 + \gamma_1 \ V \ .$$

The yaw at a distance s is

$$\delta = \frac{1}{nV} \int_{s_0}^{s} \frac{F\alpha(l - e)}{I_1 V} \sin n \ (s - u) \ du \ ,$$

which may be written

$$\delta = \frac{1}{nV} \int_{s_0}^{s} \frac{F}{I_1} \frac{(l - e)}{V} \beta_1 \ \sin n \ (s - u) \ du + \frac{1}{nV} \int_{s_0}^{s} \frac{F\gamma_1(l - e)}{I_1} \sin n \ (s - u) du.$$

The yaw due to a constant cross-wind w_1 is

$$\delta = \frac{w_1}{V} \{1 - \cos n \ (s - s_0)\}$$

Thus if we assume the thrust to be constant, the yaw due to the variable equivalent nozzle offset α is equal to the yaw due to a constant nozzle offset β_1 added to the yaw due to a constant cross wind of strength w_1 where

$$w_1 = \frac{F}{I_1 n^2} \gamma_1(l - e) = \frac{F}{I_1 n^2} (l - e) \frac{\gamma}{f} = W(l - e)\gamma/_{I_1 n^2}$$

W being the total mass of the rocket. Thus if λ_1 is the deviation of the rocket in flight due to a nozzle offset of 10^{-3} radians and λ_2 that to an unit cross-wind, the deviation due to the variable nozzle offset α is

$$\bar{\lambda} = (\beta - \gamma V_0/f)\ \lambda_1 \times 10^3 + W\ (l - e)\ \gamma\lambda_2/I_1 n^2\ .$$

For the 3-inch rocket with parallel sided shell, A.D. fuze and tubular charge the values of λ_1 and λ_2 are 0.8 degs. and 0.12 degs. respectively. Thus the deviation of this rocket with wall thickness variation specified by $b = 10^{-3}$ ins. is 1.0 degs.

In all the above work it has been assumed that the elastic limit is not exceeded, that is, Hooke's law is valid apart from the variation of Young's modulus with temperature. The problem of thermal stresses in 3-inch and 2-inch rocket tubes has been dealt with in detail by Oldroyd (Ref. 3). From his analysis he concludes that in firings at air temperature, serious excesses of the calculated stresses above the elastic limit occur near the surfaces of the tube wall but the major part of the maeterial of the tube is not stressed beyond this limit. The highest stresses occur towards the venturi end of the combustion chamber and are compressive stresses near the inner surface and tensile stresses near the outer surface of the tube wall. Beyond the elastic limit the value of Young's modulus decreases very rapidly with increasing strain and also we may expect that the coefficient of thermal expansion increases rapidly. The expression for T given by equation 3.4 is a measure of the variability of kET around the section. If we exceed the elastic limit then clearly very much larger variations of kE are possible and this we may expect to have a very serious effect on bending. The stress system is exceedingly complex and no theoretical calculations are possible; in any case the variability depends to a marked extent on the residual stresses in the tube.

The effect of thermal stresses is best illustrated by comparing tubes of different wall thickness. The standard 3-inch tubes considered by Oldroyd are 0.080 ins. thick. We consider also tubes of the same internal diameter but with wall thickness 0.20 ins. The temperatures in the thick tubes have been calculated using the same emissivity coefficients as used in calculating the temperatures of the standard tubes so that the results are strictly comparative. The temperatures are calculated for uncoated tubes with tubular charges and the venturi end of the combustion chamber.

Tube Temperatures ($^{\circ}$C.)

	0.08 in. tube		0.20 in. tube	
t (secs.)	Inner surface	Outer surface	Inner surface	Outer surface
0.2	520	260	490	8
0.4	660	460	580	70
0.6	760	580	630	150
0.8	830	670	-	-

The thermal stresses are calculated using a simplified version of Oldroyd's treatment for both wall thicknesses. We make two assumptions, (i) there is cylindrical symmetry about the rocket axis, (ii) if v is the displacement in the axial direction, we assume $\frac{\partial v}{\partial x}$ is constant through the wall of the tube. Both these conditions are approximately true.

If u is the displacement in the radial direction i.e. an element originally at a radius r is displaced to r + u, the classical equations of elasticity take the form

$$R = \frac{2G}{m-2}\left[(m-1)\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial v}{\partial x} - (m+1)\,kT\right],$$

$$H = \frac{2G}{m-2}\left[\frac{\partial u}{\partial r} + (m-1)\frac{u}{r} + \frac{\partial v}{\partial x} - (m+1)\,kT\right],$$

$$X = \frac{2G}{m-2}\left[\frac{\partial u}{\partial r} + \frac{u}{r} + (m-1)\frac{\partial v}{\partial x} - (m+1)\,kT\right].$$

G being the shearing modulus, m the inverse Poisson's ratio and R, H and X are the radial, hoop and axial stresses respectively. From the condition of equilibrium between these stresses we have

$$H - R = r\frac{\partial R}{\partial r}.$$

From this condition we find

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(ur)\right) = \frac{m+1}{m-1}\frac{\partial Tk}{\partial r}.$$

On integration we have

$$\frac{1}{r}\frac{\partial}{\partial r}(ur) = \frac{m+1}{m-1}kT + C_0,$$

$$ur = \frac{m+1}{m-1}\int r\,k\,T\,d\,r + C_0 r^2/2 + C_1.$$

The constants C_0 and C_1 are obtained from the conditions that $R = -P$ (the internal pressure) at the inner radius and $R = 0$ at the outer radius and thus we obtain the expressions for R, H and X.

The thermal stresses given in the following tables are calculated on the assumption that Hooke's law is valid, it being assumed that the elastic limit is not exceeded, the values of Young's modulus being appropriate to the temperature of each element.

Hoop stresses at nozzle end of the tube (tons per sq.in)

t (secs.)	0.08 in. tube		0.20 in. tube	
	Inner surface	Outer surface	Inner surface	Outer surface
0.2	-52.0	31.0	-100	32.0
0.4	-40.4	26.1	-108	45.1
0.6	-28.0	20.3	-107	48.7

+ sign denotes Tension, - sign denotes Compression.

The following table gives the distribution of stress through the wall of the 0.20 in. tube at t = 0.4 secs.

Distance from inner surface (ins.)	Hoop Stress (tons/sq.in.)	Longitudinal Stress (tons/sq.in.)
0	-108	-110.0
0.02	-70.9	-73.5
0.04	-45.1	-42.7
0.06	-15.4	-18.0
0.08	+4.1	+ 1.5
0.10	+18.9	+16.3
0.12	+33.4	+30.8
0.14	+37.2	+34.6
0.16	+41.9	+39.3
0.18	+44.4	+41.8
0.20	+45.1	+42.6

The following table gives the strength properties of the rocket steel at normal temperatures.

	Longitudinal (t.s.i.)	Transverse (t.s.i.)
Elastic limit	26	21
0.1% Proof Stress	38	37
0.2% Proof Stress	40	40
U.T.S.	44	44

It is seen that the calculated stresses are markedly in excess of the elastic limit with both 0.08 in. and 0.20 in. tubes and clearly with both tubes there is plastic straining, the effect with the 0.20 in. tube being very markedly greater. The figures apply at the venturi end of the tube. At the middle section of the tube the calculated stresses are approximately half those given above and with 0.08 in. tube there is only slight plastic deformation at this section. With the 0.20 in. tube the plastic deformation is still considerable at the middle section. With coated 0.08 in. tubes the calculated stresses exceed the elastic limit so an appreciable extent only near the venturi end but with the coated 0.2 in. tube the elastic limit is exceeded appreciably beyond the middle section of the tube.

APPENDIX I

Calculation of the heating of the rocket tube in the case of star centre charges which are sealed at the head end

We make the assumption that on ignition, propellant gases flow round the charge to pressurise the annular space between charge and tube. Flow is assumed to cease and then interchange of heat between the stagnant gases and the tube wall tubes place. The charge coating is assumed to absorb negligible heat.

The following constants are taken for the gas

Pressure = 70 atmospheres
Universal gas constant, R, = 0.08204 litre atmospheres/^{o}C.
Gram molecules/grm. n = 0.0424 moles/grm.
Specific heat of gas at constant pressure, C_p = 0.369 cals/grm.
Isobaric flame temperature degrees above ambient T_p = 2400 centigrade degrees
Also;- The gap between propellant coating and wall d = 0.2 cms.
Density of steel, q = 7.85 grm/cc.
Specific heat of steel, s = 0.12 cals/grm.
Thickness of steel tube, t = 0.2 cms.

We calculate the specific volume, V,

$$V = \frac{nRT_p}{P} = \frac{.0424 \times .08204 \times 2400 \text{ litre/grm.}}{70}$$

$$V = 120 \text{ cc/grm.}$$

Heat content of the gas per square cm. of surface $= \dfrac{d\,c_p\,T_p}{V}$

$$= \frac{0.2 \times 0.369 \times 2400}{120} = 1.476 \text{ calories/sq.cm.}$$

If the equilibrium temperature is T, then we have

$$\frac{d\,C_p\,T_p}{V} = \frac{d\,C_p\,T}{V} + t\,q\,s\,T$$

$$\frac{0.2 \times 0.369 \times 2400}{120} = \frac{0.2 \times 0.369}{120} + 0.2 \times 7.85 \times 0.12\,T$$

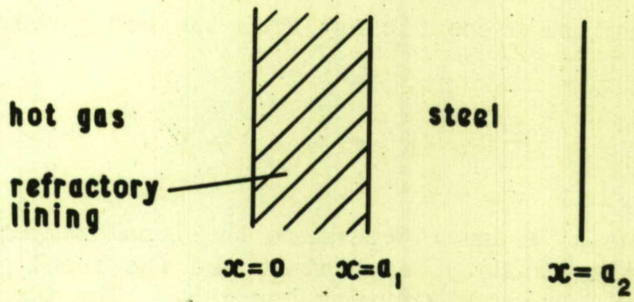
from which $T = 7.8^{o}$C.

In fact, the flow will cause the tube to heat to a higher temperature at the source end, and lower at the sealed end, but the average temperature rise along the tube should be about 7.8^{o}C. Some complications occur due to gas dissolving in the charge restrictive coating causing more gas to heat the tube than calculated above. The distance d varies round the circumference and some asymmetric heating arises.

APPENDIX II

The effect of variations of the wall thickness and coating thickness on the temperature of the tube

It can be shown that the transfer of heat through the tube during the burning period is almost entirely radial and there is negligible conduction around the circumference. The wall thickness is small compared with the radius of the tube and thus the problem is virtually one dimensional.



We use the following notation

θ is the local temperature

$x = o$ is the interface between the coating at the gas,

$x = a_1$ is the interface between the coating and the steel tube,

$x = a_2$ is the outer surface of the tube,

s_1, ρ_1, k_1, specific heat, density and thermal conductivity of the coating

s_2, ρ_2, k_2, specific heat, density and thermal conductivity of the steel.

$$K_1 = \frac{k_1}{s_1 \rho_1} \quad , \quad K_2 = \frac{k_2}{s_2 \rho_2}$$

Z is the temperature of the gas.

The equation of heat conduction in the coating is

$$\frac{\partial}{\partial x} \left(k_1 \frac{\partial \theta}{\partial x}\right) = \frac{\partial}{\partial t} (s_1 \rho_1 \theta)$$

and in the steel

$$\frac{\partial}{\partial x} \left(k_2 \frac{\partial \theta}{\partial x}\right) = \frac{\partial}{\partial t} (s_2 \rho_2 \theta)$$

During the period of burning there is negligible heat transfer from the outer surface of the steel thus

$$\frac{\partial \theta}{\partial x} = o \ , \ x = a_2 \qquad \text{(i)}$$

At $x = a_1$ the temperature and of course the heat flow are continuous thus

$$\theta_{a_1 - \epsilon} = \theta_{a_1 + \epsilon} \qquad \text{(ii)}$$

$$k_1 \left(\frac{\partial\theta}{\partial x}\right)_{a_1 - \epsilon} = k_2 \left(\frac{\partial\theta}{\partial x}\right)_{a_1 + \epsilon} , \qquad \text{(iii)}$$

ϵ being a very small positive quantity.

At $x = o$ there is heat transfer from the hot gases to the coating and this gives the boundary condition

$$k_1 \frac{\partial\theta}{\partial x} = h_1(\theta - z) , x = o ,$$

where h_1 is the transmission coefficient from the hot gases. This is usually written

$$\frac{\partial\theta}{\partial x} = h (\theta - z) , x = o , \qquad \text{(iv)}$$

The quantities k, s and ρ depend on the local temperature and the value of depends on the boundary temperature and the local gas velocity and these quantities vary appreciably during burning. The temperatures in standard tubes have been obtained accurately by numerical methods. The amount of work involved is immense and since we are concerned with the effects due to small changes in parameters, appreciable errors may arise if we use these methods. Thus an alternative method has been adopted.

In the first instance we assign mean values to s, ρ, k and h. The differential equations thus reduce to

$$\frac{\partial^2\theta}{\partial x^2} = \frac{1}{K_1} \frac{\partial\theta}{\partial t} , o < x < a_1 , \qquad \text{(v)}$$

$$\frac{\partial^2\theta}{\partial x^2} = \frac{1}{K_1} \frac{\partial\theta}{\partial t} , a_1 < x < a_2 \qquad \text{(vi)}$$

The solution of equations (v) and (vi) subject to the boundary conditions (i) and iv) is

$$\theta = z (1 - A_1 \phi_1 e^{-\alpha_1^2 t} - A_2 \phi_2 e^{-\alpha_2^2 t} \qquad \text{(vii)}$$

where

$$\phi_n = \sin \frac{\alpha_n x}{\sqrt{K_1}} + \frac{1}{h}\frac{\alpha_n}{\sqrt{K_1}} \cos \frac{\alpha_n x}{\sqrt{K_1}} , o < x < a_1 ,$$

$$= \frac{\left(\sin \frac{\alpha_n a_1}{\sqrt{K_1}} + \frac{1}{h}\frac{\alpha_n}{\sqrt{K_1}} \cos \frac{\alpha_n a_1}{\sqrt{K_1}}\right) \cos \frac{\alpha_n (a_2 - x)}{\sqrt{K_1}}}{\cos \frac{\alpha_n (a_2 - a_1)}{\sqrt{K_2}}} \quad a_1 < x < a_2$$

and the terms α_n are the roots of the determinant

$$\begin{vmatrix} \sin \frac{\alpha a_1}{\sqrt{K_1}} + \frac{\alpha}{h\sqrt{K_1}} \cos \frac{\alpha a_1}{\sqrt{K_1}} , & \cos \frac{\alpha}{\sqrt{K_2}} (a_2 - a_1) \\ \frac{k_1}{\sqrt{K_1}} \left(\cos \frac{\alpha a_1}{\sqrt{K_1}} - \frac{\alpha}{h\sqrt{K_1}} \sin \frac{\alpha a}{\sqrt{K_1}} \right. , & \frac{k_2}{\sqrt{K_2}} \sin \frac{\alpha}{\sqrt{K_2}} (a_2 - a_1) \end{vmatrix} = 0$$

Also

$$A_n = \frac{k_1}{\alpha_n \sqrt{K_1}} \Big/ \left(s_1 \rho_1 \int_0^{a_1} \phi_n^2 \, dx + s_2 \rho_2 \int_{a_1}^{a_n} \phi_n^2 \, dx \right)$$

and
$$\int_0^{a_1} \phi_n^2 \, dx = \left[\frac{a_1}{2} - \frac{\sqrt{K_1}}{4\alpha_n} \sin \frac{2\alpha_n a_1}{\sqrt{K_1}} + \frac{\alpha_n^2}{Kh^2} \left(\frac{a_1}{2} + \frac{\sqrt{K_1}}{4\alpha_n} \sin \frac{2\alpha_n a_1}{\sqrt{K_1}} \right) \right.$$
$$\left. + \frac{1}{2h} \left(1 - \cos \frac{2\alpha_1 a_1}{\sqrt{K_1}} \right) \right] ,$$

$$\int_{a_1}^{a_2} \phi_n^2 \, dx = \frac{\left(\sin \frac{\alpha_n a_1}{\sqrt{K_1}} + \frac{\alpha_2}{h\sqrt{K_1}} \cos \frac{\alpha a_1}{\sqrt{K_1}} \right)^2}{\left[\cos \frac{\alpha (a_2 - a_1)}{\sqrt{K_2}} \right]^2} \left[\frac{a_2 - a_1}{2} + \frac{\sqrt{K_2}}{4\alpha_n} \sin \frac{2\alpha (a_2 - a_1)}{\sqrt{K_2}} \right]$$

In all these expressions α_n is denoted by α

In the absence of the coating the solution is given by equation (vii) but with

$$\phi_n = \sin \alpha_n x + \frac{\alpha_n}{h} \cos \alpha_2 x$$

$$A_n = - 2Z \Big/ \left(1 + \frac{\alpha_n^2}{h^2}\right)(a\alpha_2 + \sin \alpha_n a_1 \cos \alpha_2 a)$$

In the first instance we take

$$s_1 = 0.2 \quad \rho_1 = 1.96 \quad k_1 = 0.0036$$
$$s_2 = 0.135 \quad \rho_2 = 7.7 \quad k_2 = 0.098$$

these being in C.G.S and ^{o}C units. The mean thickness of the coating is 0.006 ins. and the mean wall thickness 0.08 ins.

Three values of h for the coated tubes have been used in the calculation namely h = 10, 25 and 75 and these cover the range of h in practise.

We proceed thus. For each value of h we calculate the mean tube temperature for three values of $a_2 - a_1$, namely the value 0.203 cms (0.08 ins.) and two other values 0.185 cms and 0.220 cms. Let $(\theta_m)_1$ and $(\theta_m)_2$ be these two latter values and L the wall thickness, thus

$$\left(\frac{\partial \theta_2}{\partial L}\right)_{L = 0.203} = \left[(\theta_m)_2 - (\theta_m)_1\right] \Big/ (0.220 - 0.185) .$$

In addition we calculate θ_1 and θ_2 the temperatures at $x = a_1$ and $x = a_2$ with $a_2 - a_1 = 0.203$. It is found that at the three values of h the term

$$\lambda = 2\left(\frac{\partial \theta_m}{\partial L}\right) \Big/ (\theta_1 + \theta_2)$$

is practically independent of the time and of h.

Coated tubes

	λ		
t (secs.)	h = 10	h = 25	h = 75
0.15	3.97 *	3.97 *	3.97 *
0.25	4.34	4.35	4.35
0.35	4.50	4.47	4.45
0.45	4.54	4.48	4.36
0.55	4.55	4.46	4.34
0.65	4.54	4.42	4.22
0.75	4.55	4.35	4.14
0.85	4.50	4.34	4.04
0.95	4.45	4.26	3.94
1.05	4.39	4.21	3.83

*The series expressions converge slowly at t = 0.15 and these may be in slight error.

The calculations have been repeated with the uncoated tube with h = 0.5, h = 1, h = 2, these covering the ranges of h obtained in practise.

Uncoated tubes

	λ		
t (secs.)	h = 0.5	h = 1.0	h = 2.0
0.15	3.95	4.12	4.16
0.25	4.23	4.30	4.34
0.35	4.44	4.36	4.34
0.45	4.49	4.38	4.24
0.55	4.47	4.33	4.13
0.75	4.42	4.22	3.84
0.95	4.38	4.08	3.55
1.05	4.34	3.92	3.26

We are concerned with the first 0.6 secs. of burning and in this region λ has a constant value for both coated and uncoated tubes. The reason for this constancy of λ is apparent. Let us suppose that amount of heat crossing the boundary into the tube is independent of small variations of the wall thickness. Thus the mean temperature of the tube is

$$\theta_m = \frac{H}{Ls_2\rho_2}$$

and if we assume $s_2\rho_2$ as constant where

$$\frac{\partial\theta_m}{\partial L} = \frac{-H}{L^2 s_2\rho_2} = \frac{1}{L}\theta_m$$

or

$$\frac{1}{\theta_m}\frac{\partial\theta_m}{\partial L} = -\frac{1}{L}$$

We have L = 0.203 so that 1/L = 4.9. The value $(\theta_1 + \theta_2)/2$ is approximately equal to θ_m and thus we see that the constancy of λ is due to the condition that the heat that has previously crossed the boundary varies only slightly with small variations in the wall thickness. The temperature θ_1 and θ_2 have been used as reference temperatures since these have already been computed accurately.

Repeat calculations have shown that λ is invariant to small changes in k, and as we may expect and thus we can confidently use the formula:-

$$\frac{\partial\theta_m}{\partial L} = \frac{\lambda(\theta_1 + \theta_2)}{2}$$

and the value of λ taken is 4.4 in centimetre units or 11.2 when the wall thickness is measured in inches. This is correct for both coated and uncoated tubes.

The values of θ_1 and θ_2 have been computed accurately in earlier works (Ref. 1) and the results are given in Table I.

APPENDIX III

Bending of the motor in static firing

In Section 2 we consider the bending of the motor tube in flight. There it is noted that the natural frequency of bending is 87 cycles per second. In the static firing the head end of the motor tube is rigidly clamped and this changes the natural frequency. We have seen in Section 2 that for a perfect rocket, the pressure enters only in the gas whip terms and provided we take the values of Young's modulus appropriate to normal temperatures, this affects the amplitude terms having only a secondary effect on the frequency. Further the thrust has only a small effect on the frequency under these conditions. The detailed problem is considered in a later report. Neglecting the pressure and thrust terms the differential equation becomes

$$D^4 y + \lambda^2 \frac{\partial^2 y}{\partial t^2} = 0 \;, \; \lambda^2 = \frac{w}{EI} \;, \; D = \frac{\partial}{\partial x} \;, \qquad (1)$$

w being the weight per unit length of the shaft. With sufficient accuracy we may neglect the length of the venturi and assume the venturi assembly to be a point mass acting at its centre of gravity. The boundary condition thus become

$$EI\, D^3 y = W \frac{\partial^2 y}{\partial t^2} \;, \; x = l \qquad (2)$$

$$D^3 y = 0 \qquad , \; x = l \qquad (3)$$

W being the mass of the venturi assembly and l the distance of its centre of gravity from the clamp. In addition the boundary conditions at the clamp are

$$y = Dy = 0 \qquad , \; x = 0$$

Writing $y = Ye^{ipt}$, the general solution of the differential equation satisfying the conditions (4) is

$$Y = A\left[\text{Sinh } z_1 - \sin z_1\right] + B\left[\text{Cosh } z_1 - \cos z_1\right],$$

where

$$z_1 = (p\lambda)^{\frac{1}{2}} x$$

Substituting this into the boundary conditions (2) and (3) and using the notation

$$S = \text{Sinh } z \;, \; s = \sin z \;, \; C = \text{Cosh } z \;, \; c = \cos z \;, \; z = (p\lambda)^{\frac{1}{2}} l$$

we obtain the equations.

The determinant of these equations reduces to

$$1 + cC + \frac{zW}{wl}(Sc - Cs) = 0 \qquad (5)$$

This is an equation in z and W/wl. For a given value of z we calculate $\frac{W}{wl}$.

The practically important values are given in the following table

z	$\frac{wl}{W}$
0.2	0.00053
0.4	0.00854
0.6	0.0437
0.8	0.1411
1.0	0.3620
1.2	0.8263
1.3	1.230
1.4	1.841
1.5	2.833
1.6	4.57
1.8	22.62

For a specified value of $\frac{wl}{W}$ we determine z either by interpolation or graphically. The natural frequency is thus

$$\frac{1}{2\pi}\frac{z^2}{l^2}\left(\frac{EI}{w}\right)^{\frac{1}{2}} .$$

APPENDIX IV

The stability of the rocket charge

The rocket charge rests on the grid and usually it is not anchored at the head end but depends on the tube for support. In the case of tubular charges, plastic tabs placed on the charges restrict the independent motion. It is of interest to determine the stability of the charge when it is supported only at one end. Denoting the Young's modulus of the charge by E and the moment of area by I, the bending moment is

$$B = EI\,\frac{\partial^2 y}{\partial x^2} = \mu\,\frac{\partial^2 y}{\partial x^2}$$

The longitudinal force is

$$G_x = P_0 A_0 + wxf$$

P_0 being the stagnation pressure, A_0 the cross-sectional area and w the mass per unit length of the charge. The shearing stress is given by

$$S + \frac{\partial B}{\partial x} + G_x\,\frac{\partial y}{\partial x} = 0\ ,$$

and in the absence of external forces and couples the equation of motion is

$$\frac{\partial^4 y}{\partial x^2} + \frac{wfx}{\mu}\,\frac{\partial^2 y}{\partial x^2} + \frac{wf}{\mu}\,\frac{\partial y}{\partial x} + \frac{w}{\mu}\,\frac{\partial^2 y}{\partial t^2} = 0$$

Writing $\dfrac{wf}{\mu} = a\ ,\ \dfrac{\mu}{w} = c^2$ we get

$$D^4 y + aD(xDy) + \frac{1}{c^2}\,\frac{\partial^2 y}{\partial t^2} = 0$$

Let us suppose y contains t in the form $e^{\lambda ct}$; we have

$$D^4 y + aD(xDy) + \lambda^2 y = 0$$

In the first instance let us suppose the nozzle end of the charge to be rigidly connected to the tube, while the head end is free. Thus

$$y = \frac{\partial y}{\partial x} = 0\ ,\ x = l\ ;\quad B = 0\ ,\ S + G_0\,\frac{\partial y}{\partial x} = 0 \text{ at } x = 0\ ;$$

These may be written

$$y = \frac{\partial y}{\partial x} = 0 \text{ at } x = l\ ;\quad \frac{\partial^2 y}{\partial x^2} = \frac{\partial^3 y}{\partial x^3} = 0 \text{ at } x = l.$$

The solution for y may be expressed in the series form

$$y = \sum_n A_n\, x^n/_{n!}$$

Substituting this into the differential equation gives the recurrence relations for the terms A_n. All the coefficients may be expressed in terms of the first four. The boundary conditions then give four relations which A_0, A_1, A_2 and A_3 must satisfy. The determinant of these equations may be expanded in the form

$$\Delta \equiv 1 - \frac{al^3}{3!} + \frac{4}{6!}(al^3)^2 - \frac{28}{9!}(al^3)^3 + \frac{280}{12!}(al^3)^4 - \frac{3640}{15!}(al^3)^5 \ldots\ldots$$

$$+ \lambda^2 l^4 \left\{ \frac{2}{4!} - \frac{18}{7!}al^3 + \frac{200}{10!}(al^3)^2 - \frac{2670}{13!}(al^3)^3 + \frac{4.59 \times 10^4}{16!}(al^3)^4 \ldots\ldots \right.$$

$$+ \lambda^4 l^8 \left\{ \frac{8}{8!} - \frac{192}{11!}al^3 + \frac{4184}{14!}(al^3)^2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \right.$$

$$+ \lambda^6 l^{12} \left\{ \frac{3}{12!} + \ldots\ldots\ldots\ldots\ldots\ldots \right.$$

$$+ \ldots\ldots\ldots\ldots\ldots\ldots .$$

If the numerical values of a and l are substituted, the equation $\Delta = 0$ gives us the values of λ^2. If all these are negative, the charge is stable. If however one or more positive values of λ^2 are obtained, the charge is unstable. The first root λ^2 may be obtained quickly from the following approximate equation

$$\lambda_1^2 \, l^4 = -12.37 + 1.58 \, al^3 .$$

For the 3-inch rocket No. 1 tubular charge we have

$$f = 2000 \text{ f.s.}^2$$

$$w = 3.54 \text{ lb/ft.},$$

$$E = 1.6 \times 10^8 \text{ pdls./sq.ft.}$$

$$I = 1.25 \times 10^4 \text{ ft.}^4 ,$$

$$l = 3.6 \text{ ft.},$$

thus $$a\,l^3 = 16.5 .$$

The greatest stable length is such that $\lambda_1 = 0$ so that it is given by

$$1.58 \, a \, l^3 = 12.37 \text{ or } a \, l^3 = 7.84$$

thus the charge is completely unstable. The length of the charge that is just stable is 2.8 ft.

Similarly if we supposed that in addition the head end of the charge were pin pointed so that $y = B = 0$ at $x = 0$, we find the charge is still unstable.

If the charge were rigidly located at the head end so that $y = \frac{\partial y}{\partial x} = 0$ at $x = 0$ and $x = l$, the charge is almost neutrally stable.

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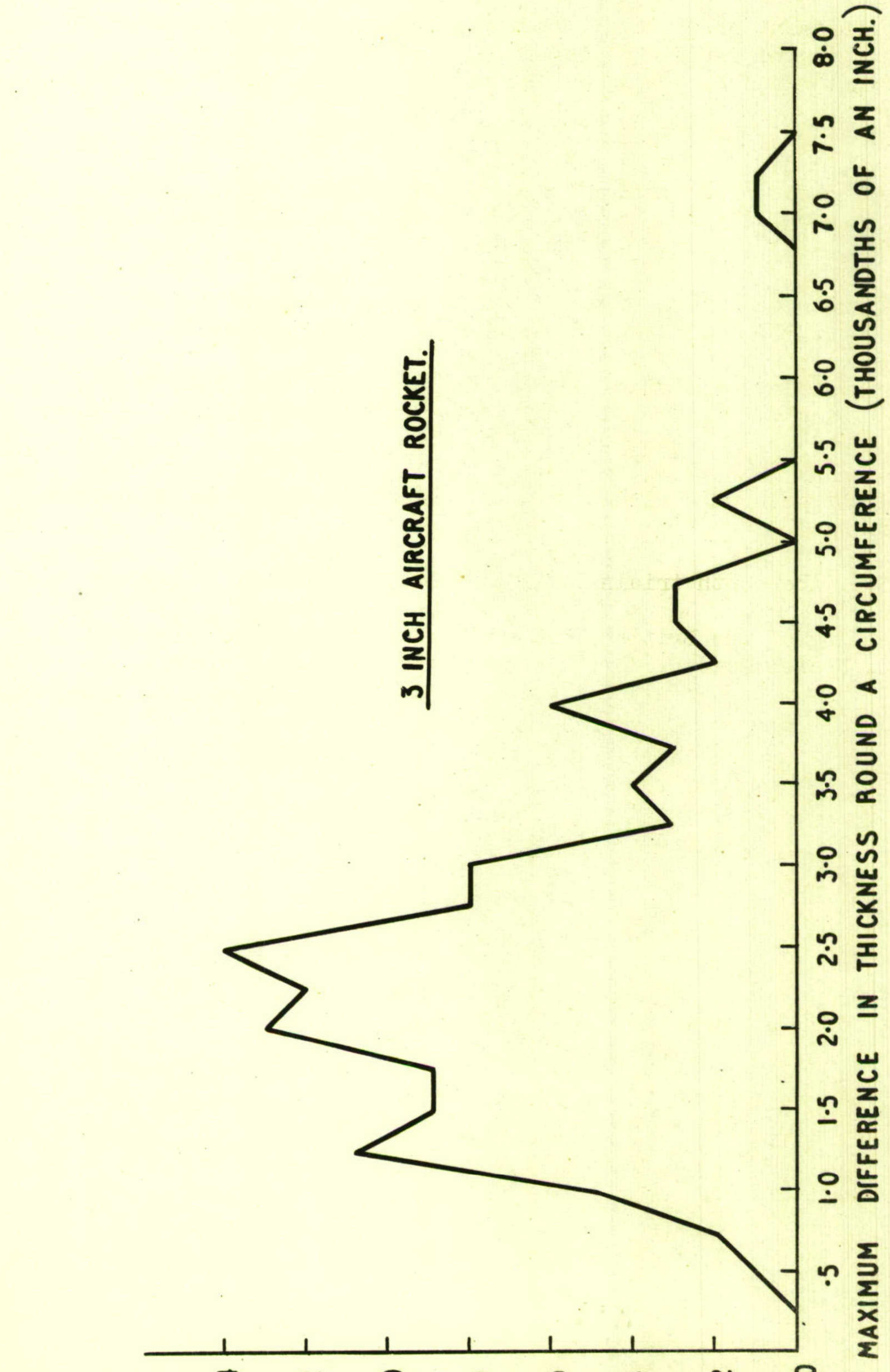

FIG 1. DISTRIBUTION OF MAXIMUM CIRCUMFERENTIAL THICKNESS VARIATION

(120 TUBES MEASURED 9·5" FROM SHELL RING END ON 8 GENERATORS 45° APART.)

FIG. 2.

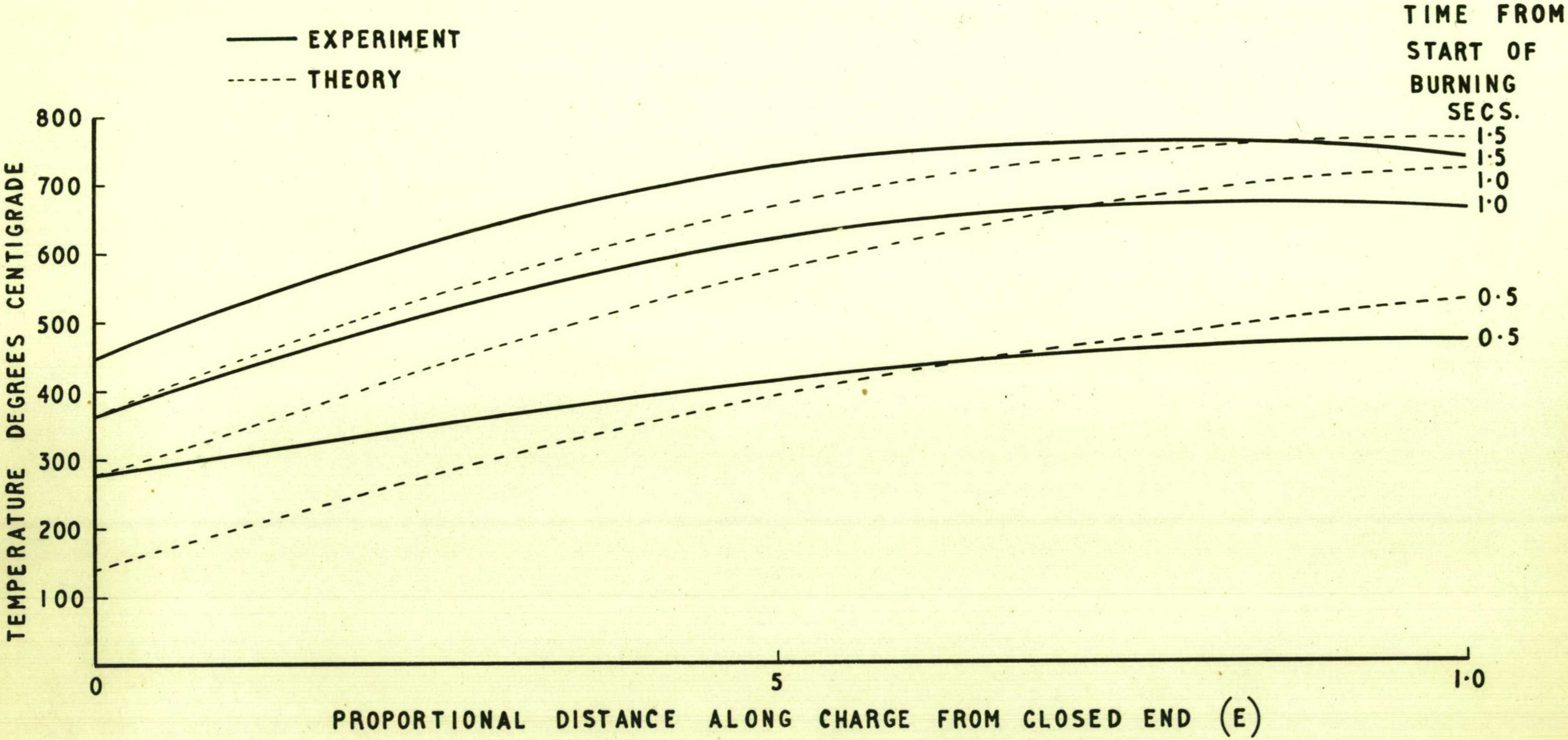

FIG. 2. 3" UNCOATED TUBE CHARGE 2·70-·0·75 SUK 43·15" LONG

TEMPERATURE DISTRIBUTION ALONG A TUBE GENERATOR.

FIG. 3.

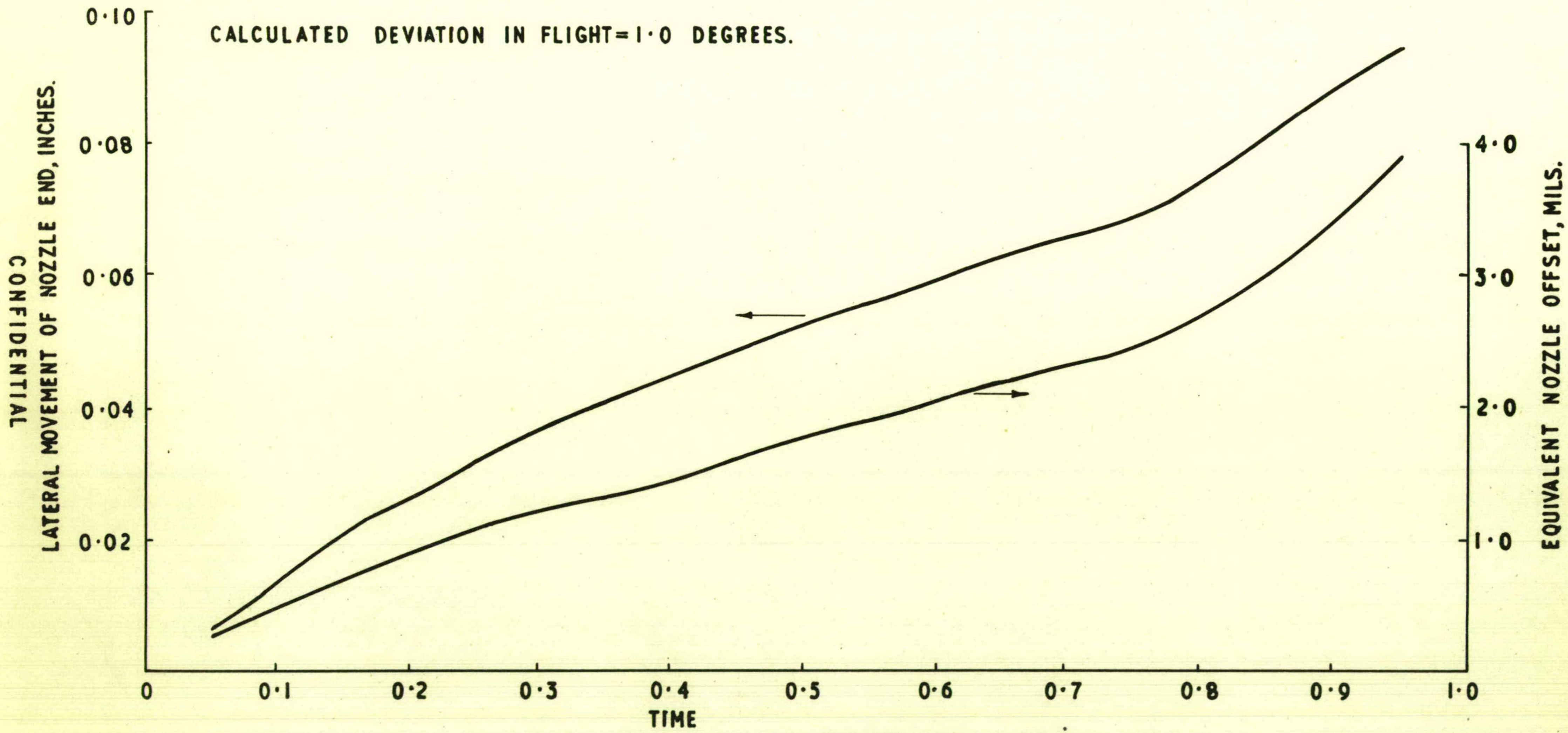

FIG. 3. BENDING OF 3 INCH ROCKET ASSUMING EVEN HEATING FROM TUBULAR CHARGE & WALL THICKNESS VARYING UNIFORMLY FROM 80–82 THOU.

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FIG. 4.

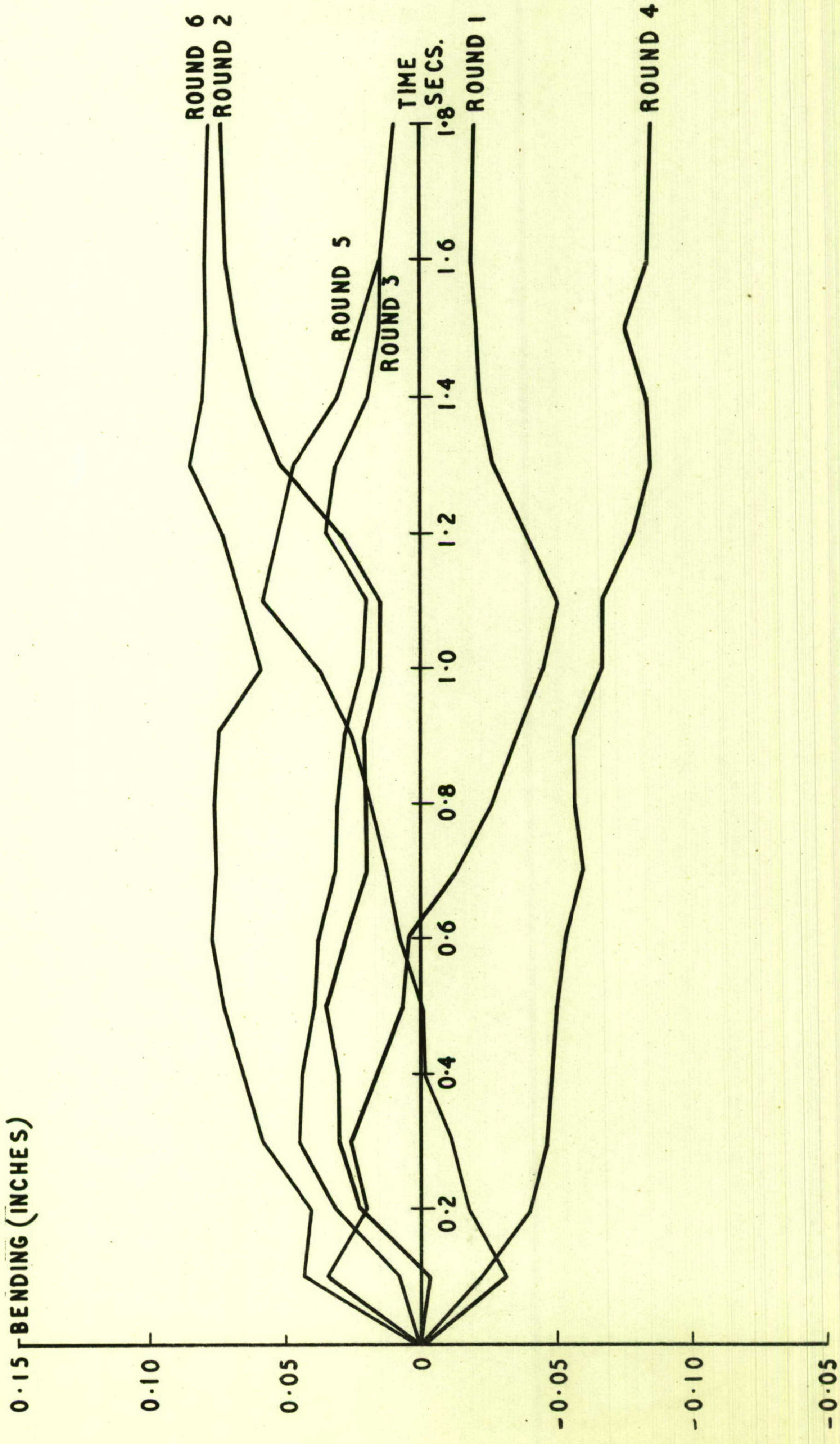

FIG. 4. 3 INCH No.1. 2·75–0·75 SUK DRILLED TUBULAR, ·008 INCH REFRACTORY LINING INTACT

FIG. 5.

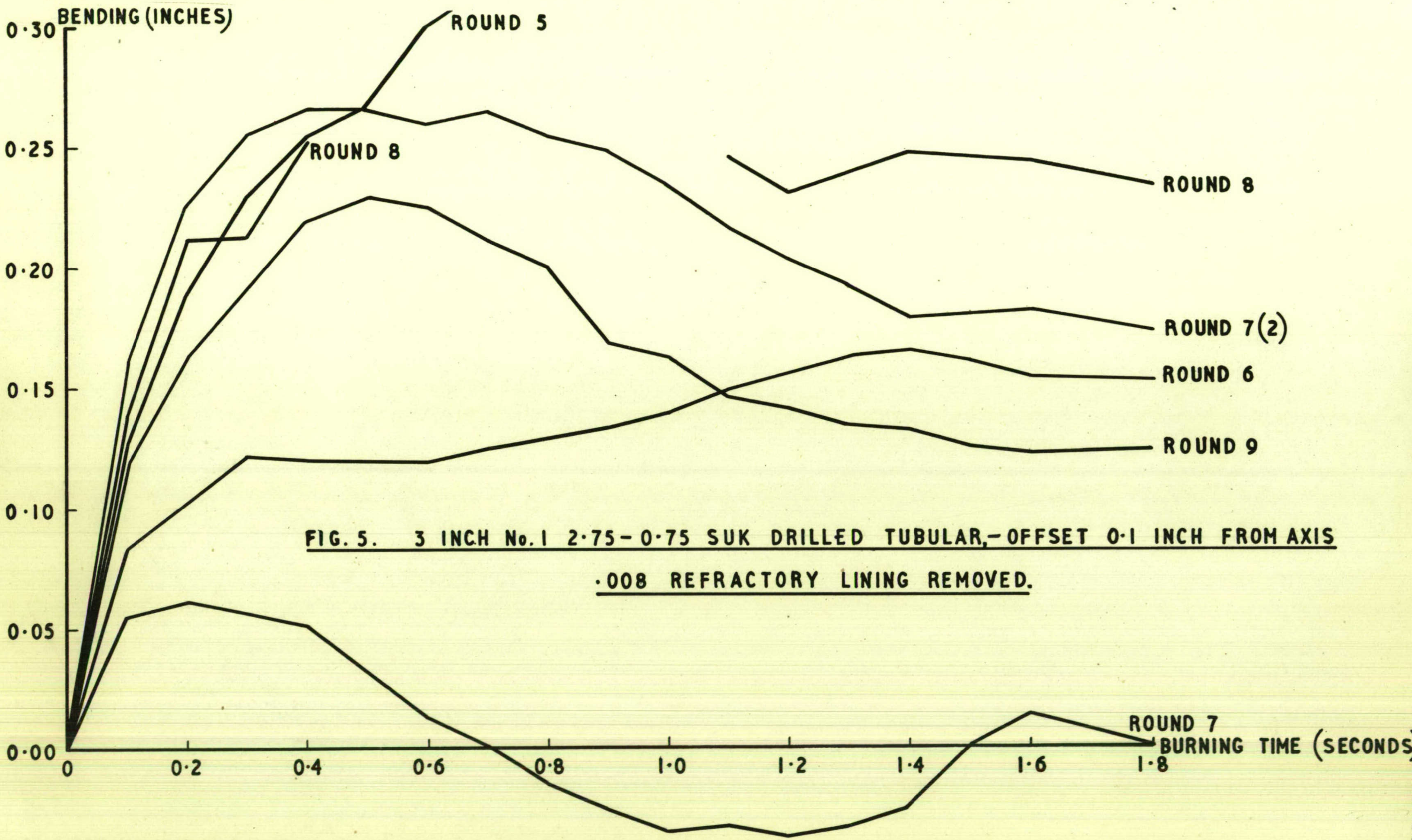

FIG. 5. 3 INCH No. 1 2·75 – 0·75 SUK DRILLED TUBULAR, – OFFSET 0·1 INCH FROM AXIS ·008 REFRACTORY LINING REMOVED.

FIG. 6.

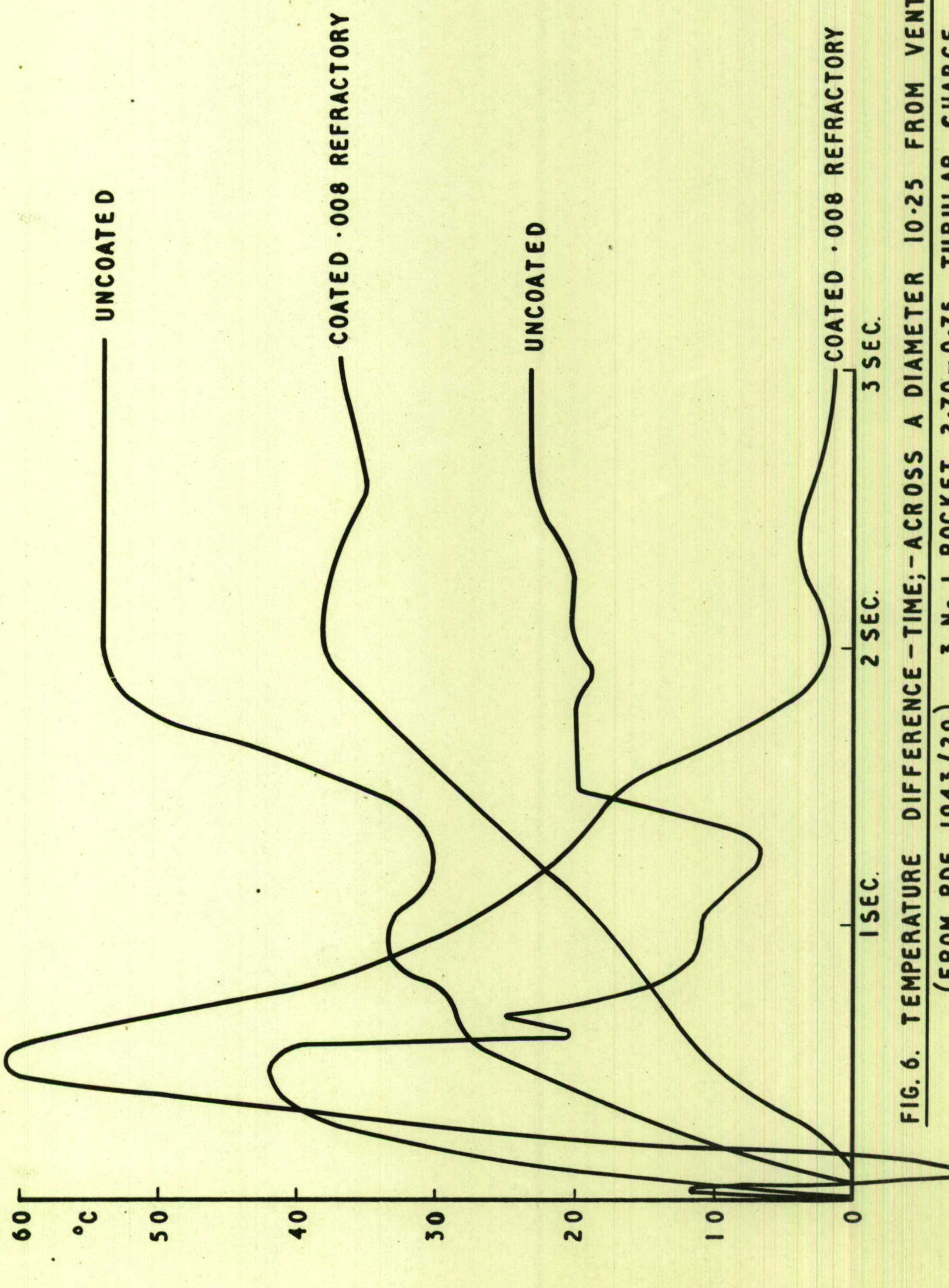

FIG. 6. TEMPERATURE DIFFERENCE – TIME; – ACROSS A DIAMETER 10·25 FROM VENTURI EXIT
(FROM PDE 1943/29) 3 No. 1. ROCKET 2·70–0·75 TUBULAR CHARGE.

FIG. 7.

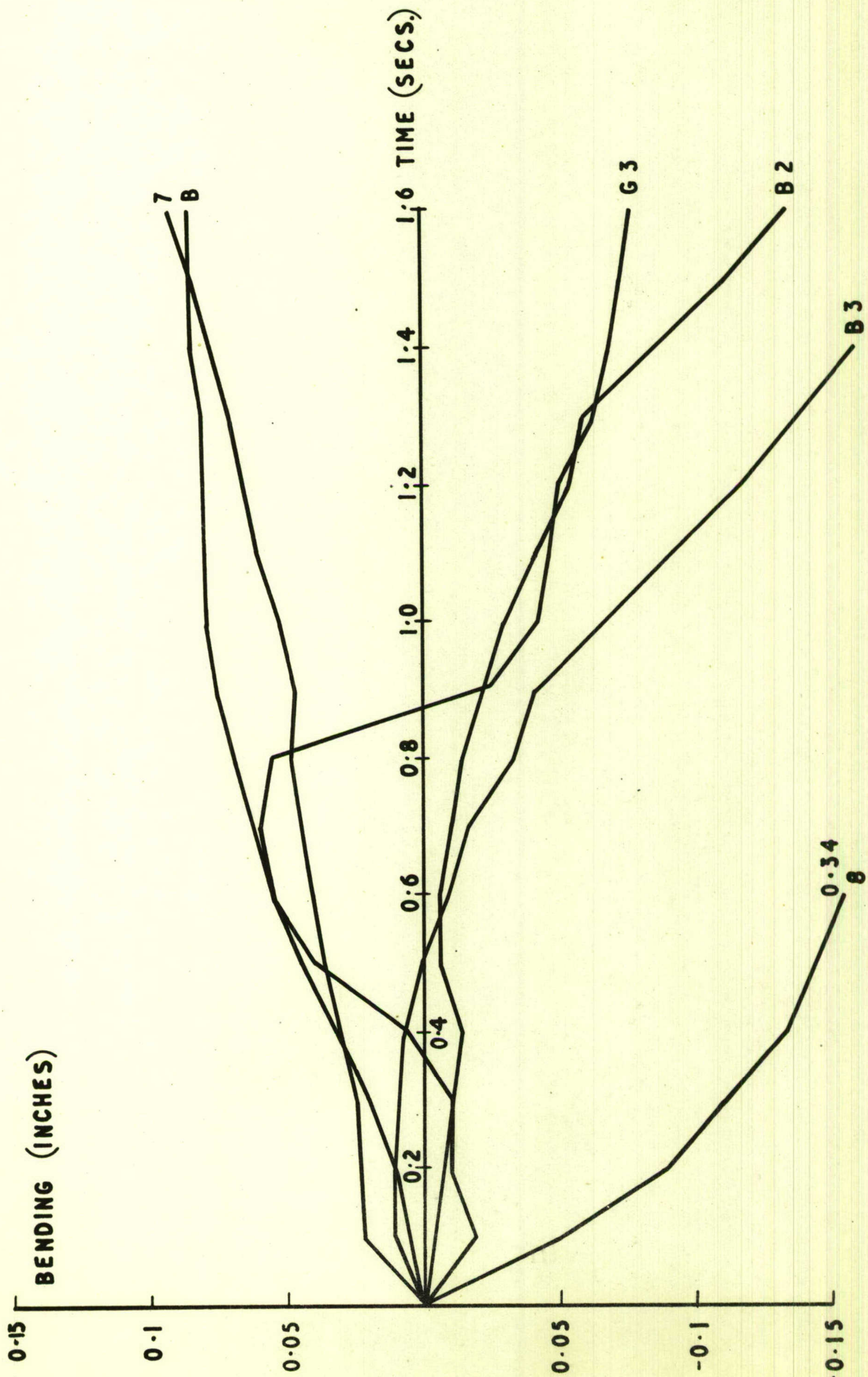

FIG. 7. 3 INCH No. 1. XII SUK, ·008 INCH REFRACTORY LINING INTACT

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FIG. 8.

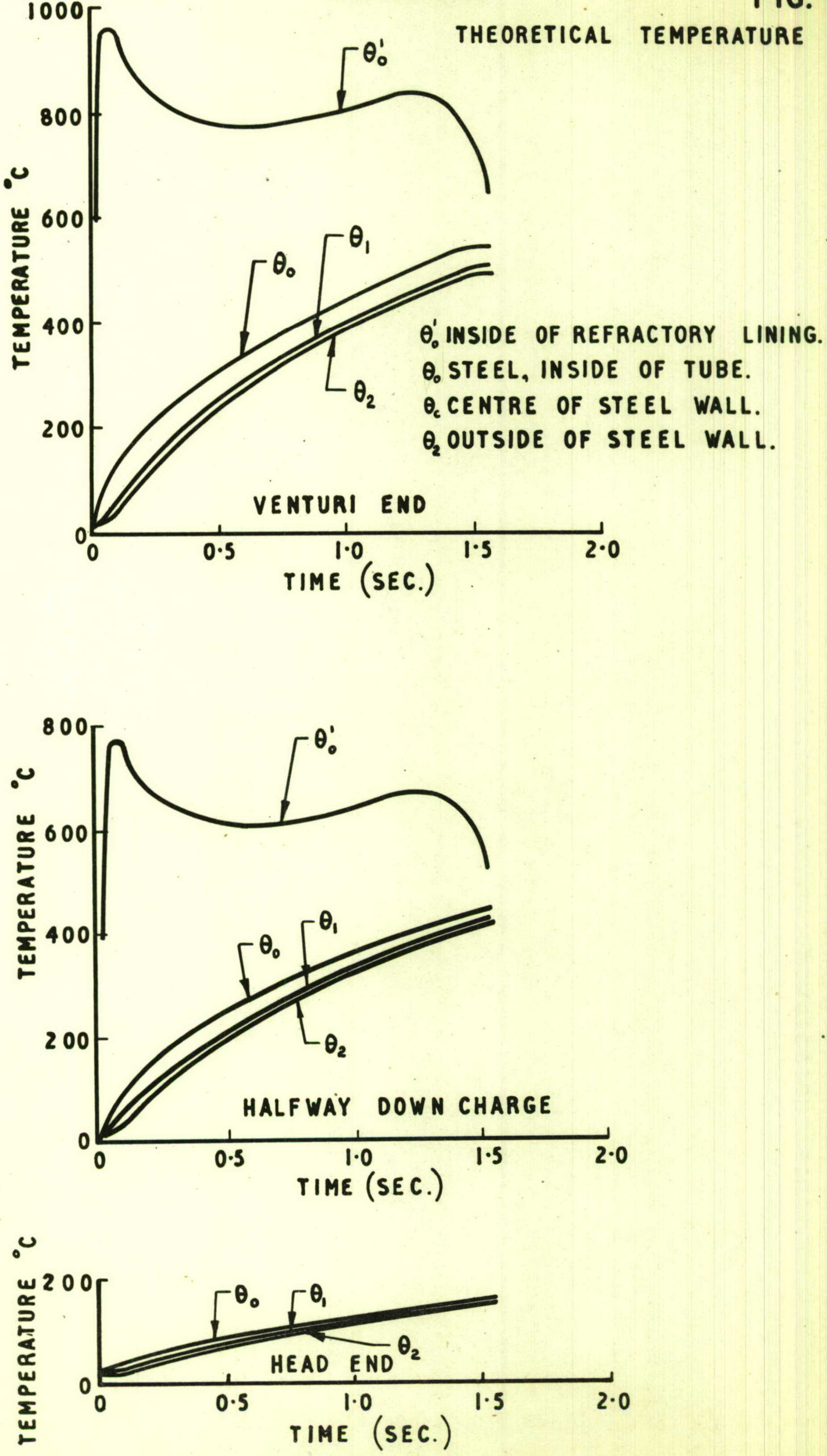

FIG. 8. 3 INCH ROCKET, COATED TUBE, FILLED SU/K/X/11.

CHARGE TEMPERATURE: 60° F (16° C.)

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FIG. 9

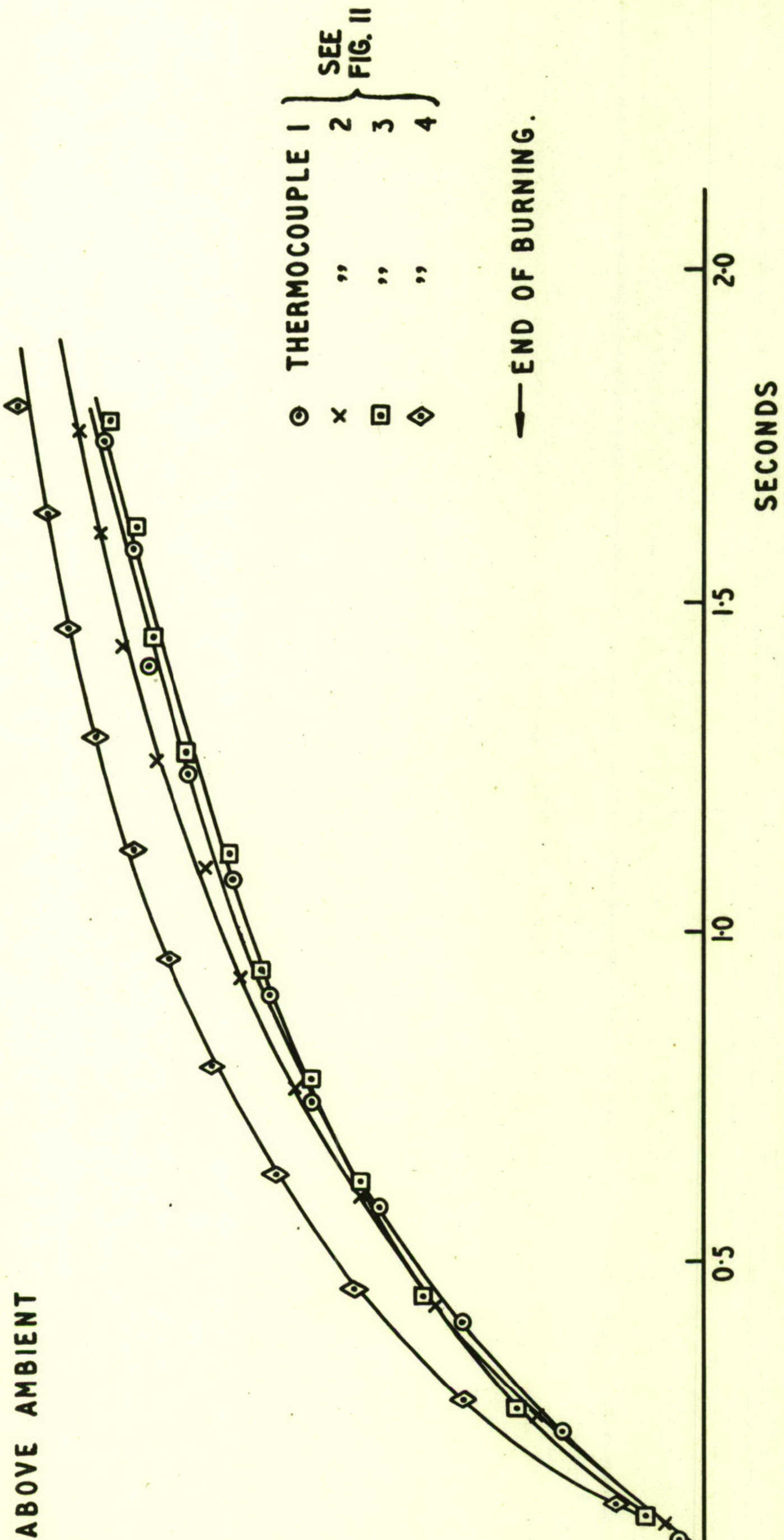

FIG. 9 3" ROCKET. NO INSULATION. CHARGE XII SUK.

TEMPERATURES ON OUTSIDE OF TUBE.

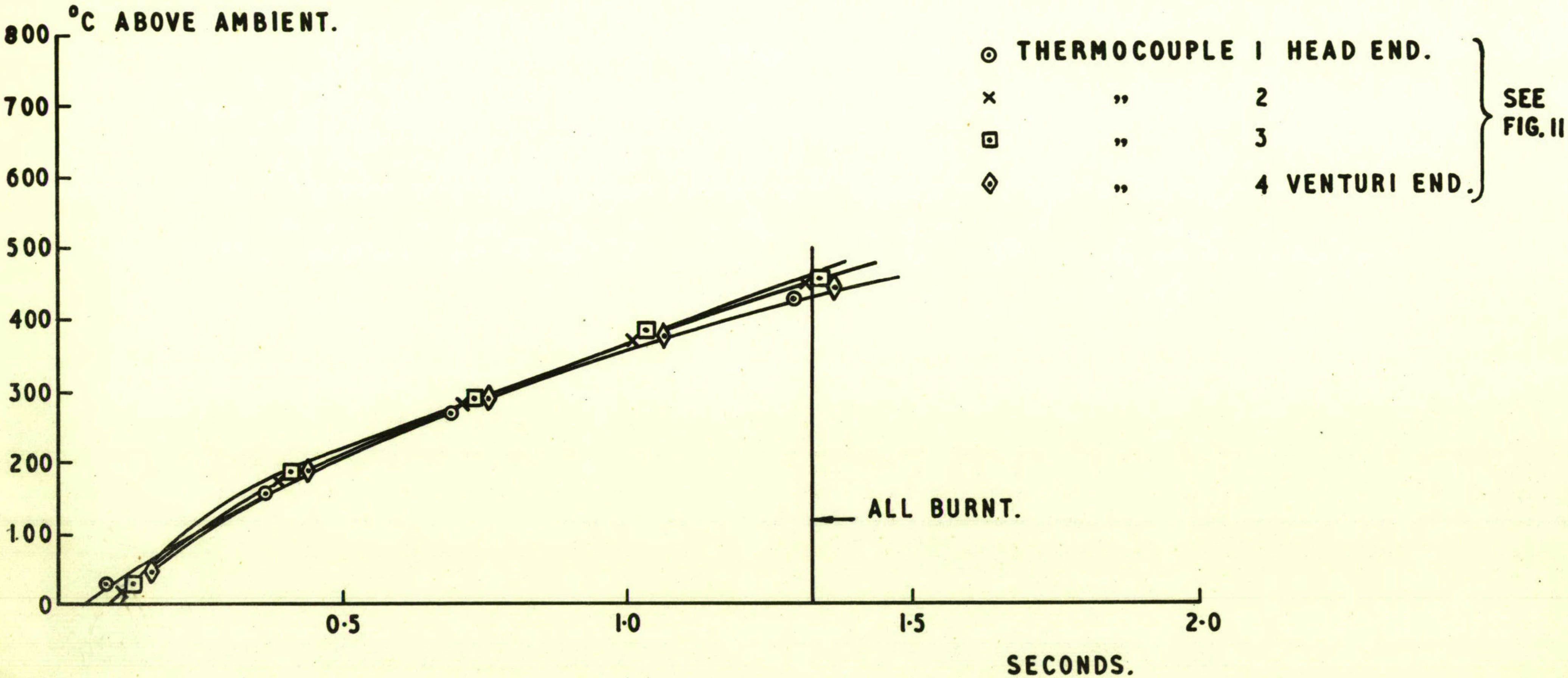

FIG. 10. 3" ROCKET 0·008" REFRACTORY LINING CHARGE XII SUK.

TEMPERATURES ON OUTSIDE OF TUBE.

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FIG. II.

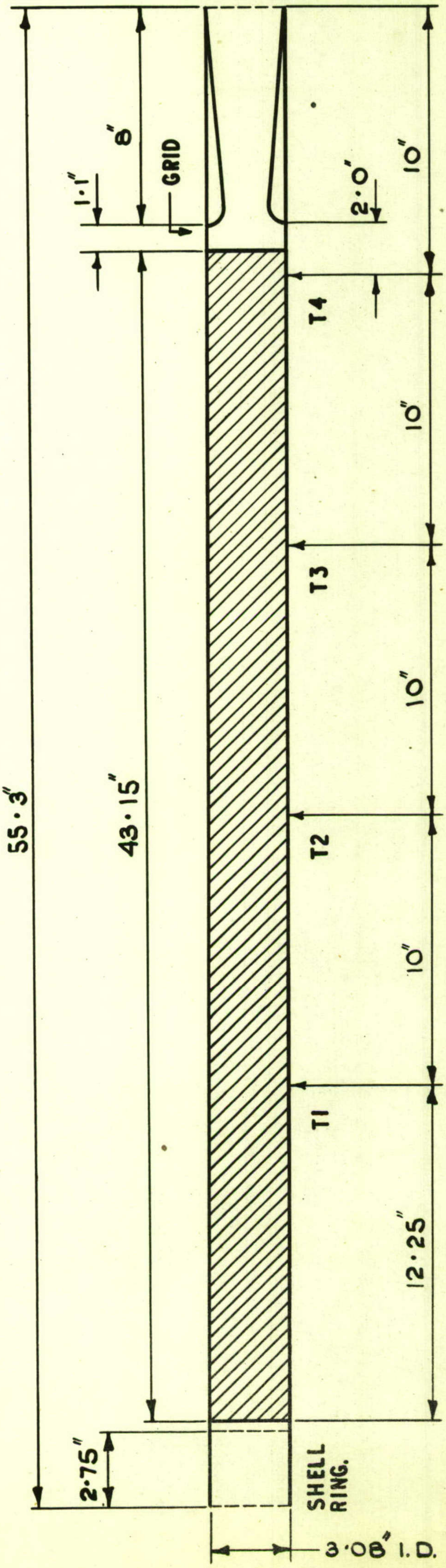

FIG. II. 3" No. I THERMOCOUPLE LOCATION.

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FIG. 12

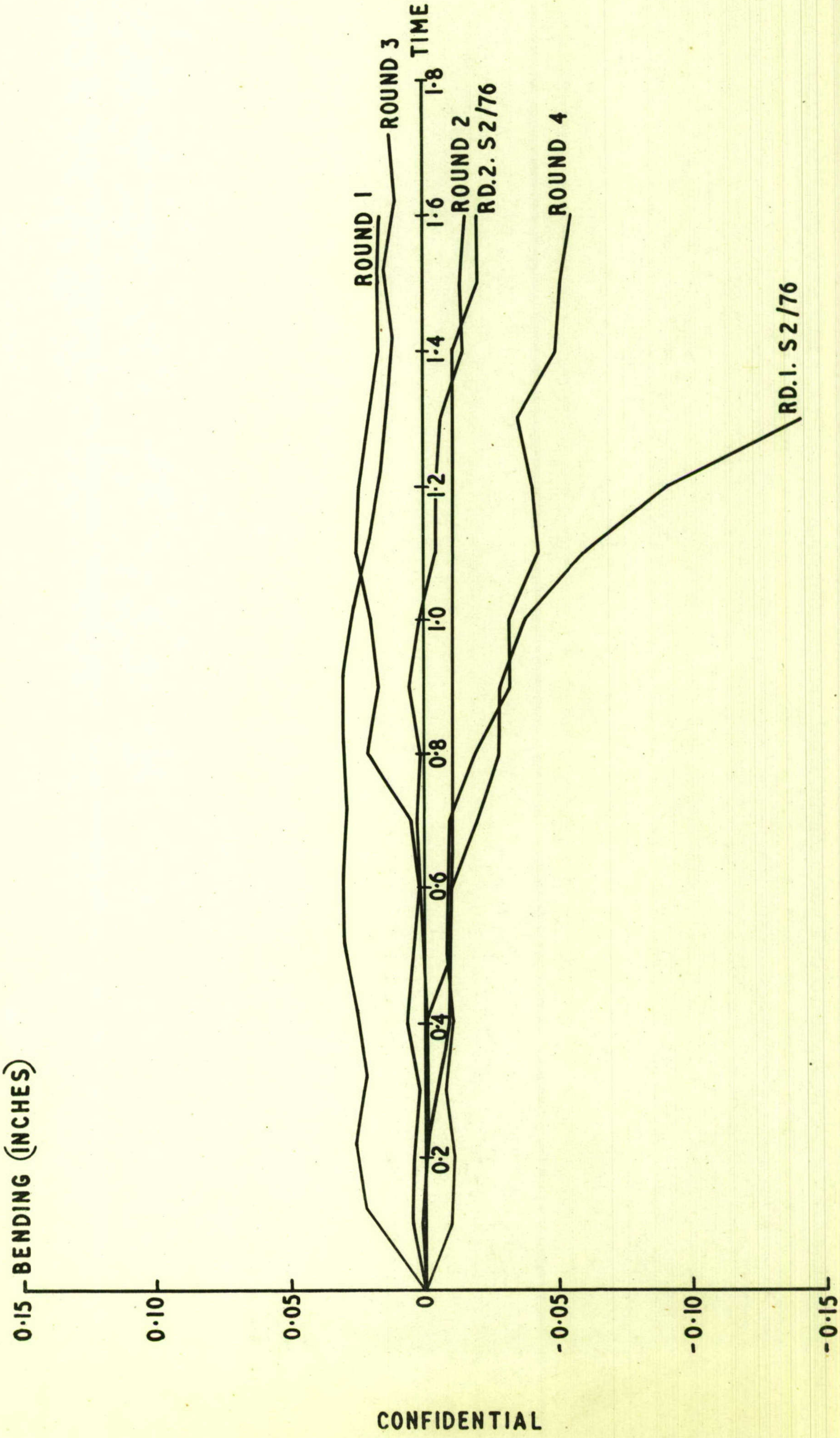

FIG. 12. 3 INCH No. 1 XII WITH 50 THOU REMOVED FROM EACH ARM (IN SUK) 50 THOU B102 NEOPRENE LINING.

FIG. 13.

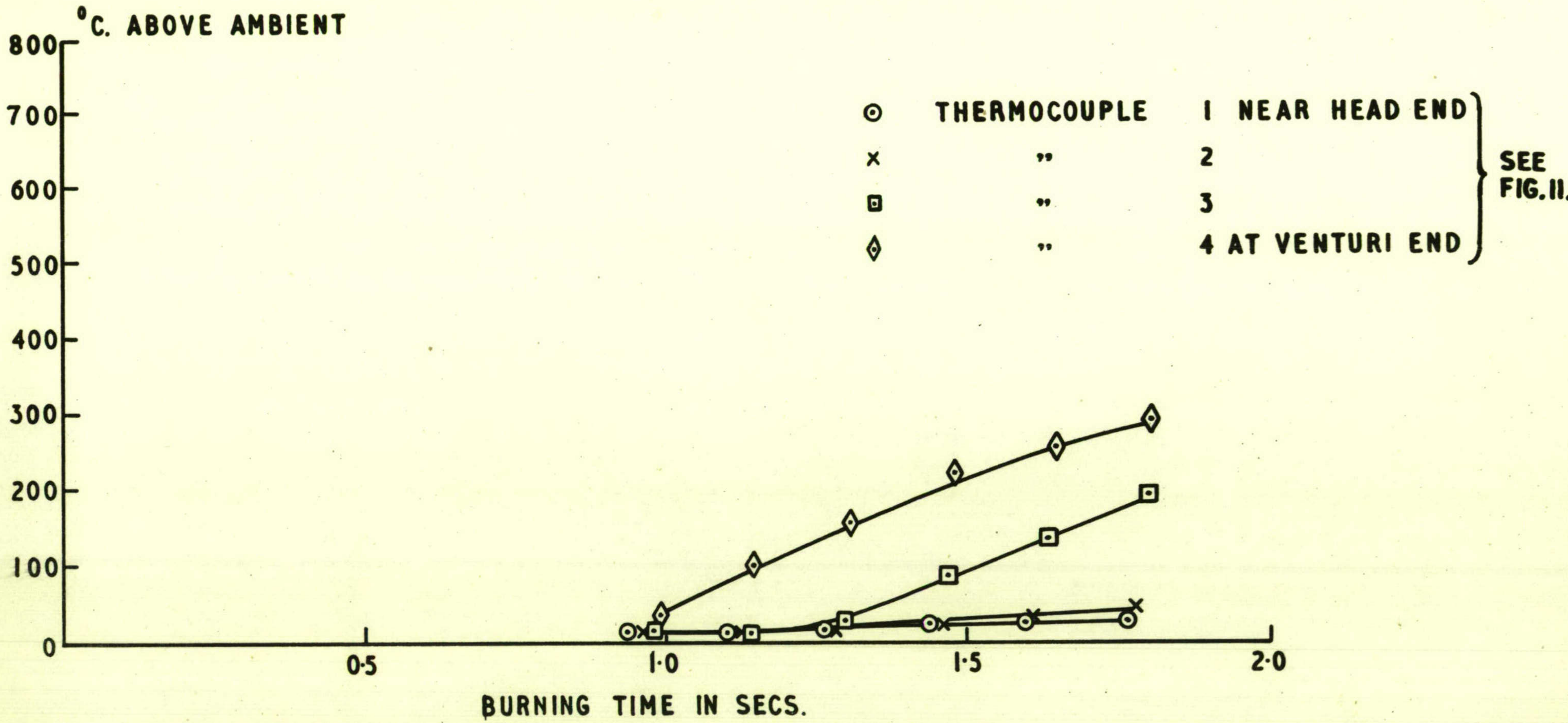

FIG. 13. 3" ROCKET 0·068 POLYTHENE LINING. CHARGE MODIFIED XII.

TEMPERATURES ON OUTSIDE OF TUBE.

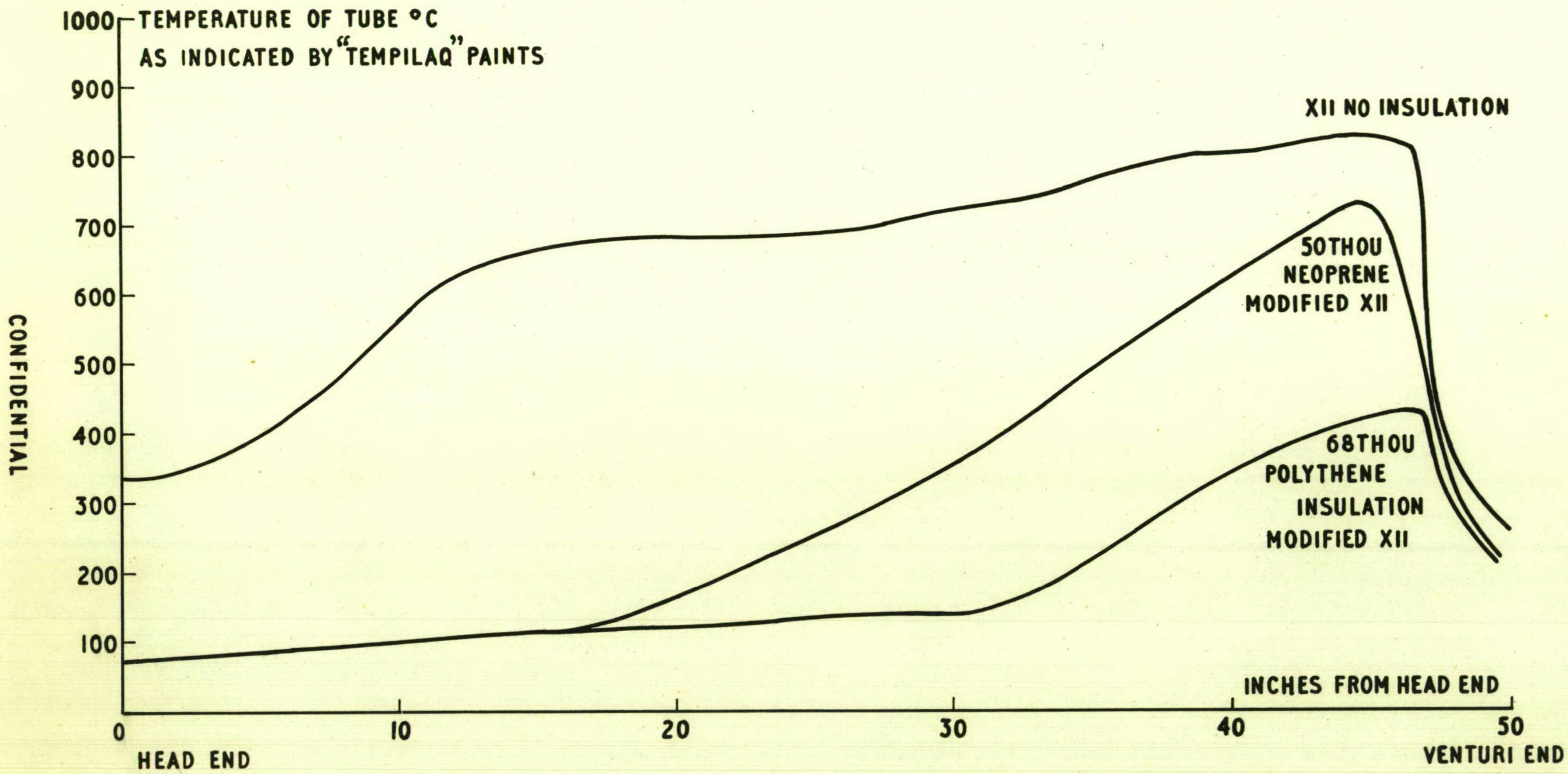

FIG.14. MAXIMUM TEMPERATURE DISTRIBUTION ON 3" No.1 ROCKETS WITH VARIOUS LININGS AND XII TYPE CHARGES.

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FIG.15.

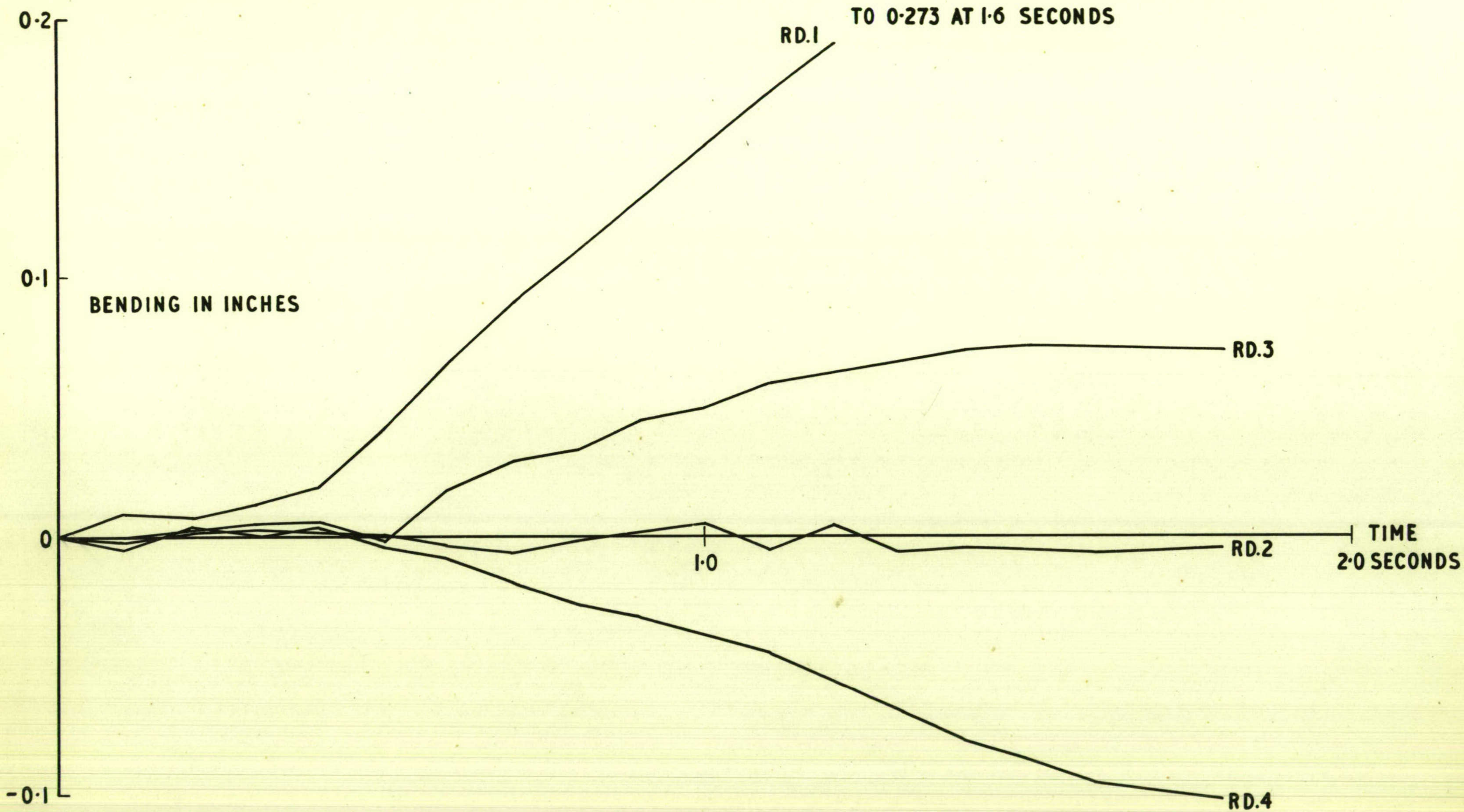

FIG.15. 3" Nº1 ROCKET. X11-SUK 50 THOU PLANED OF EACH ARM, BY16B SEAMLESS LINING 25-60 THOU.

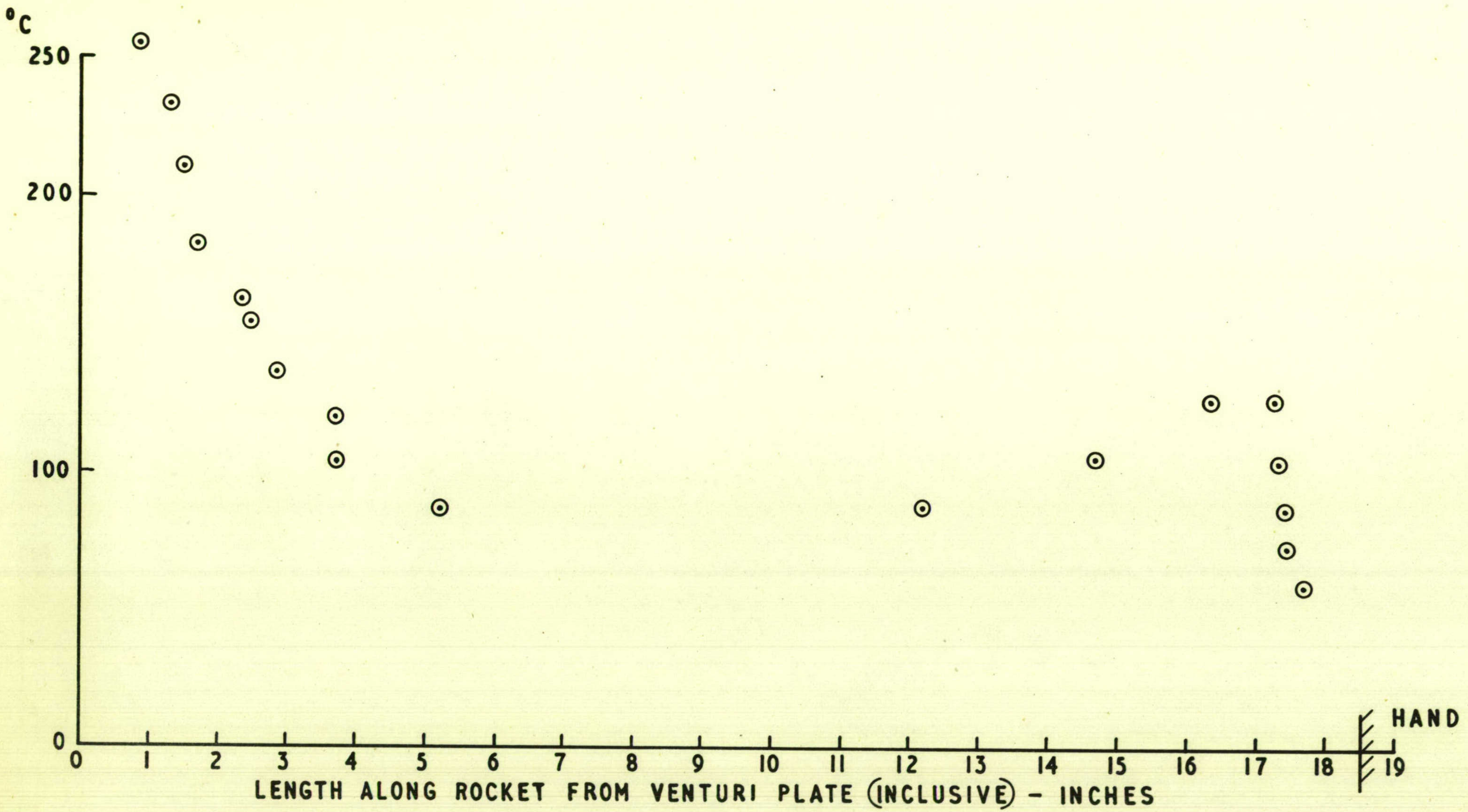

FIG. 16. MAXIMUM TEMPERATURES REACHED ON 100 MM. S.S. ROCKET FILLED PROPELLANT F.565/14/K STAR CENTRE CHARGE DURING STATIC FIRING. (AS INDICATED BY "TEMPILAQ" PAINTS)

FIG.17

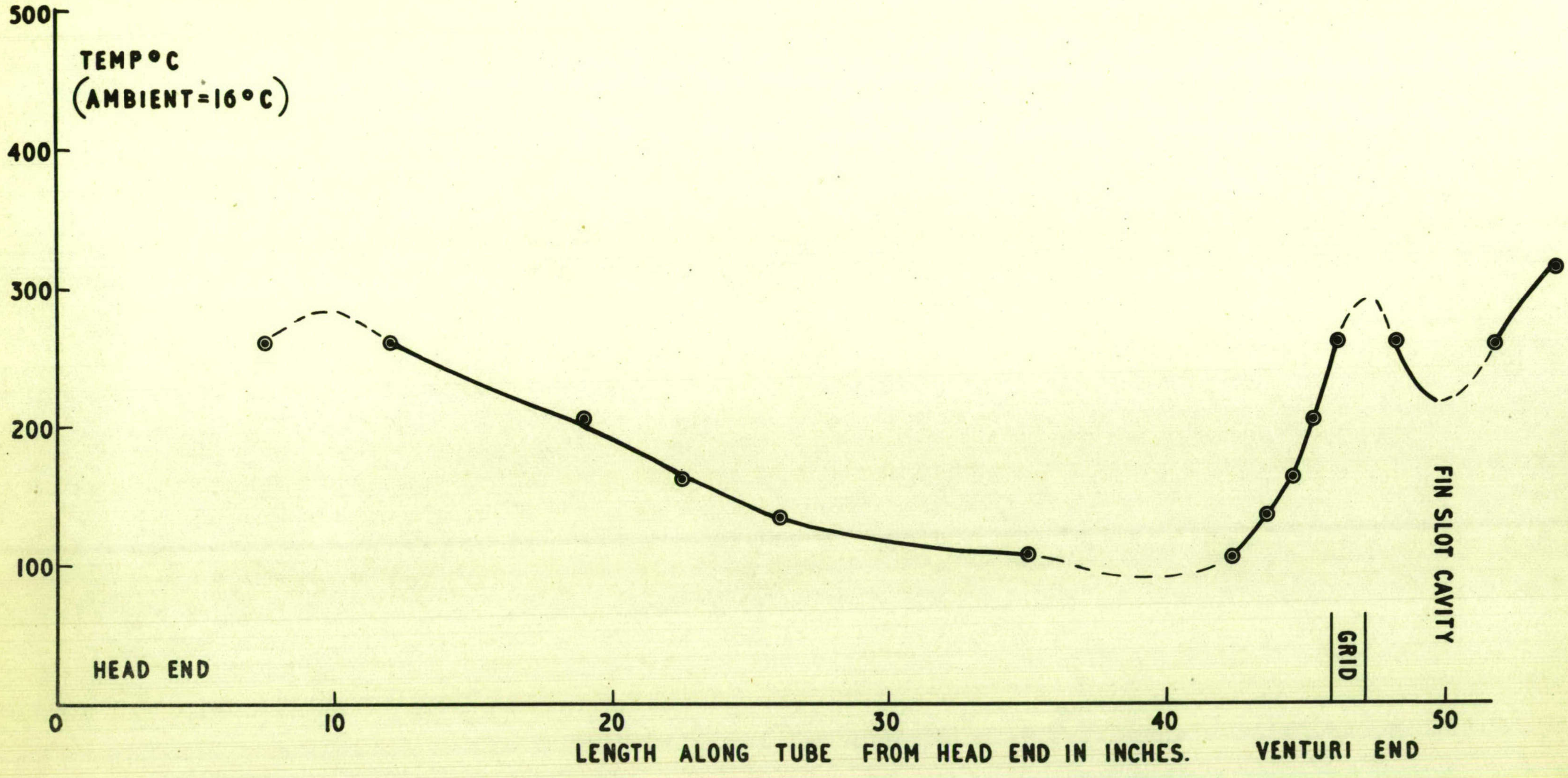

FIG. 17. 3" No.1. ROCKET WITH 43·15" x 2·9" F.B.2 / S U K STAR WITH OPEN HEAD END. DISTRIBUTION OF MAXIMUM TEMPERATURES REACHED DURING STATIC FIRING AS INDICATED BY "TEMPILAQ" PAINTS.

FIG.18.

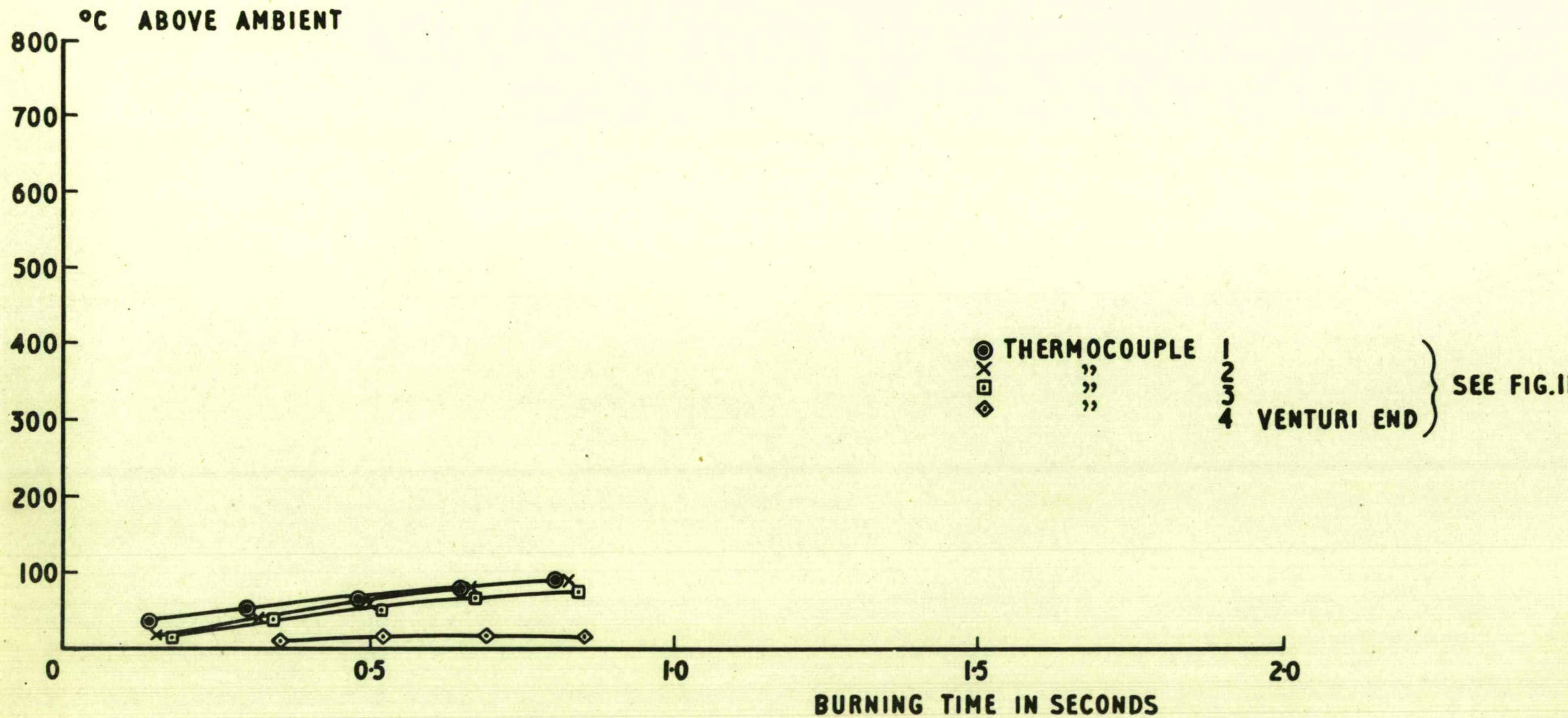

FIG.18. 3" ROCKET NO INSULATION, CHARGE FB2 STAR SUK
PLAIN OBTURATOR. HEAD UNSEALED.

FIG.19.

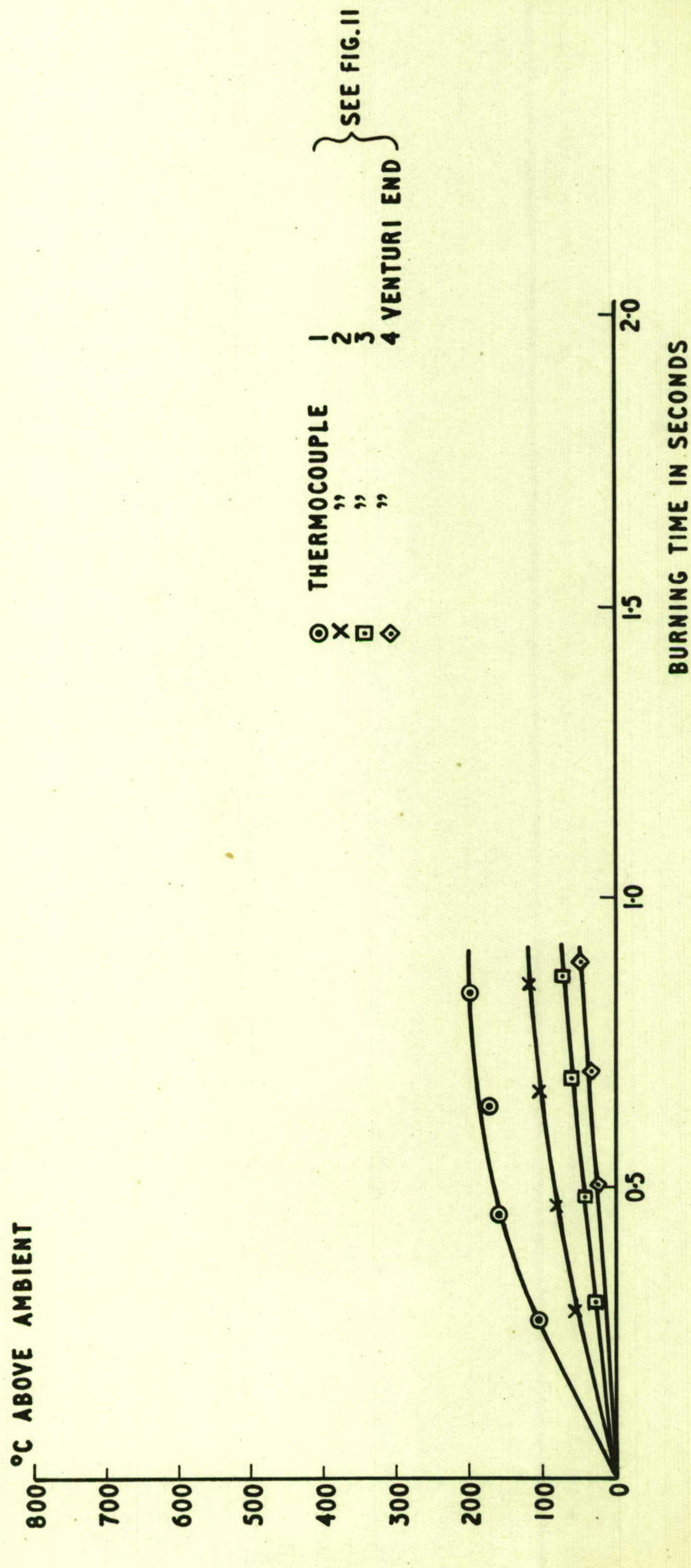

FIG.19. 3" ROCKET NO INSULATION, CHARGE FB2 STAR (SUK)
PLAIN OBTURATOR HEAD UNSEALED.

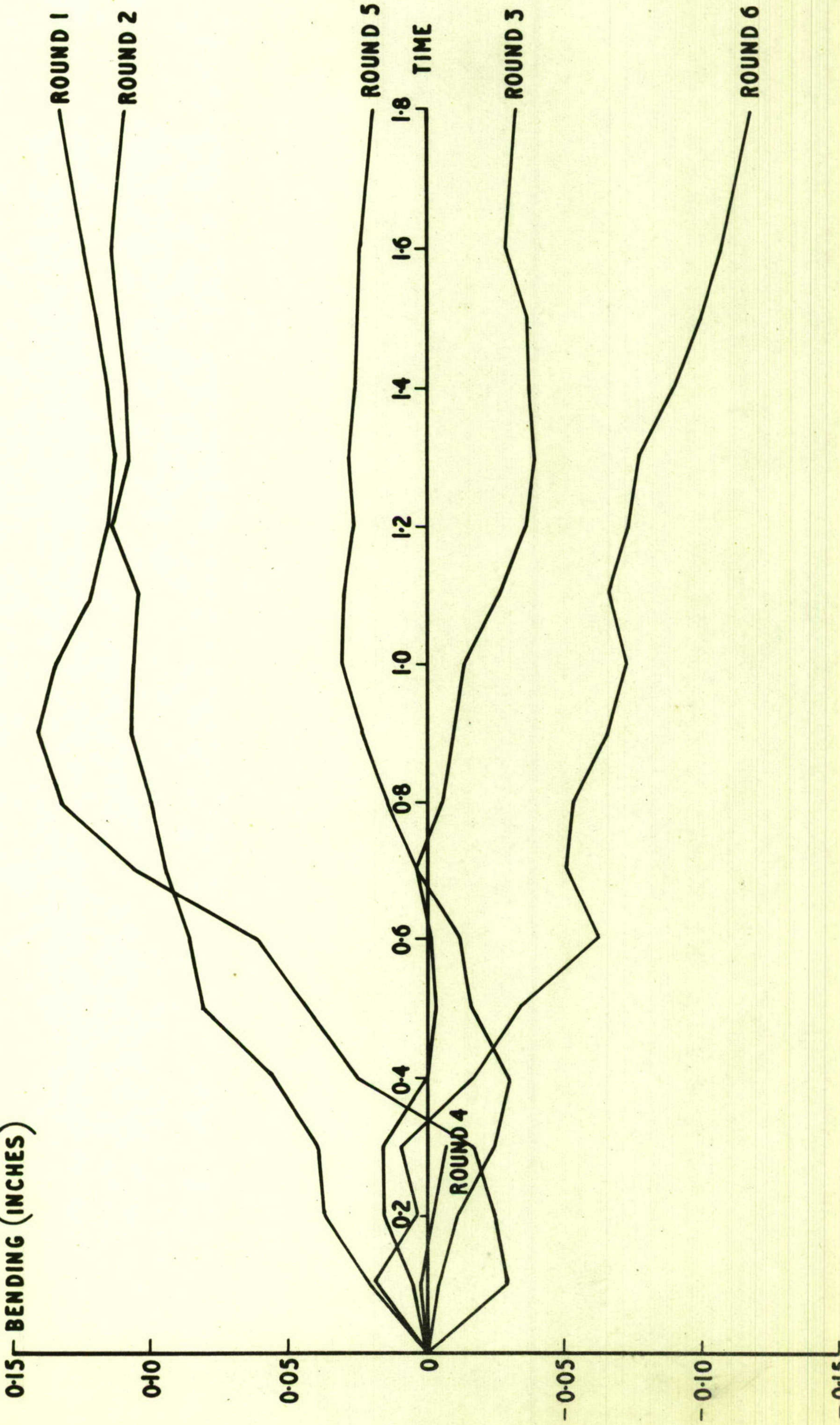

FIG.20. 3 INCH Nº1 2·9 FB2 STAR IN SUK REFRACTORY LINING INTACT AND HEAD END OF CHARGE SEALED.
(PLAIN OBTURATOR.)

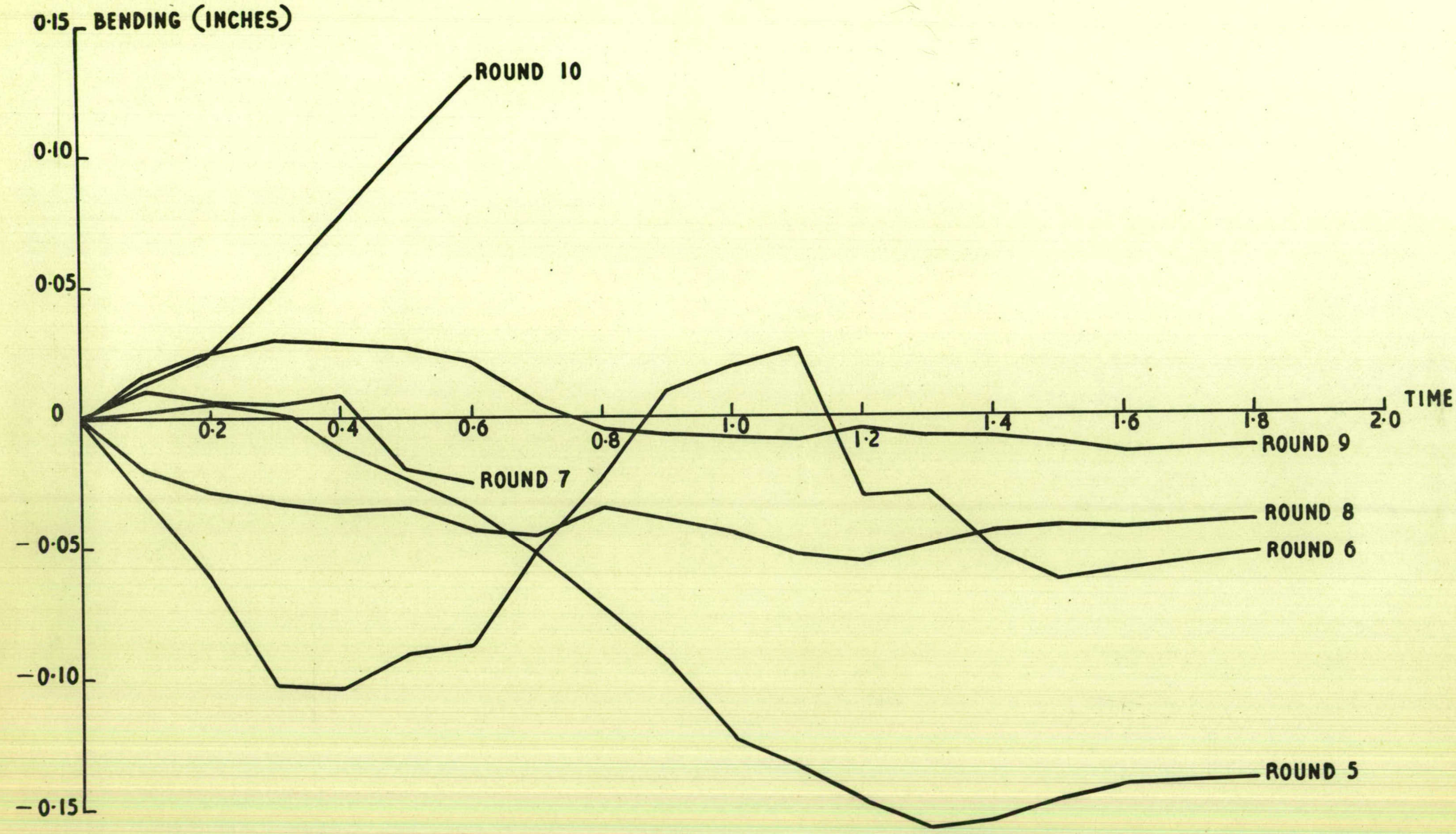

FIG. 21. 3 INCH No.1 TAPERED FB2 STAR IN 488/649 ·008 REFRACTORY LINING REMOVED. HEAD END OF CHARGE SEALED.
(PLAIN OBTURATOR)

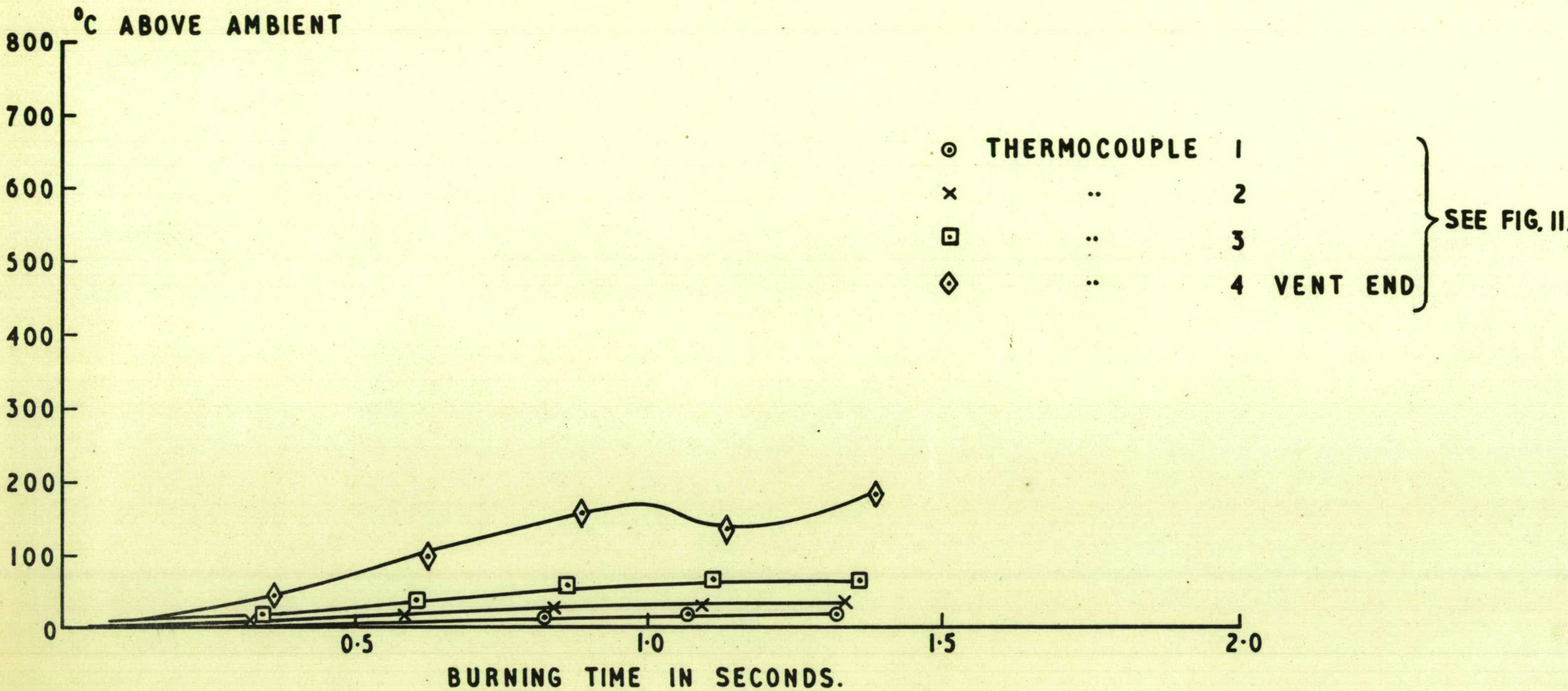

FIG. 22. 3" ROCKET 0·008" REFRACTORY LINING. CHARGE TAPERED FB2 STAR IN F488/649

PLAIN OBTURATOR. HEAD END OF CHARGE SEALED.

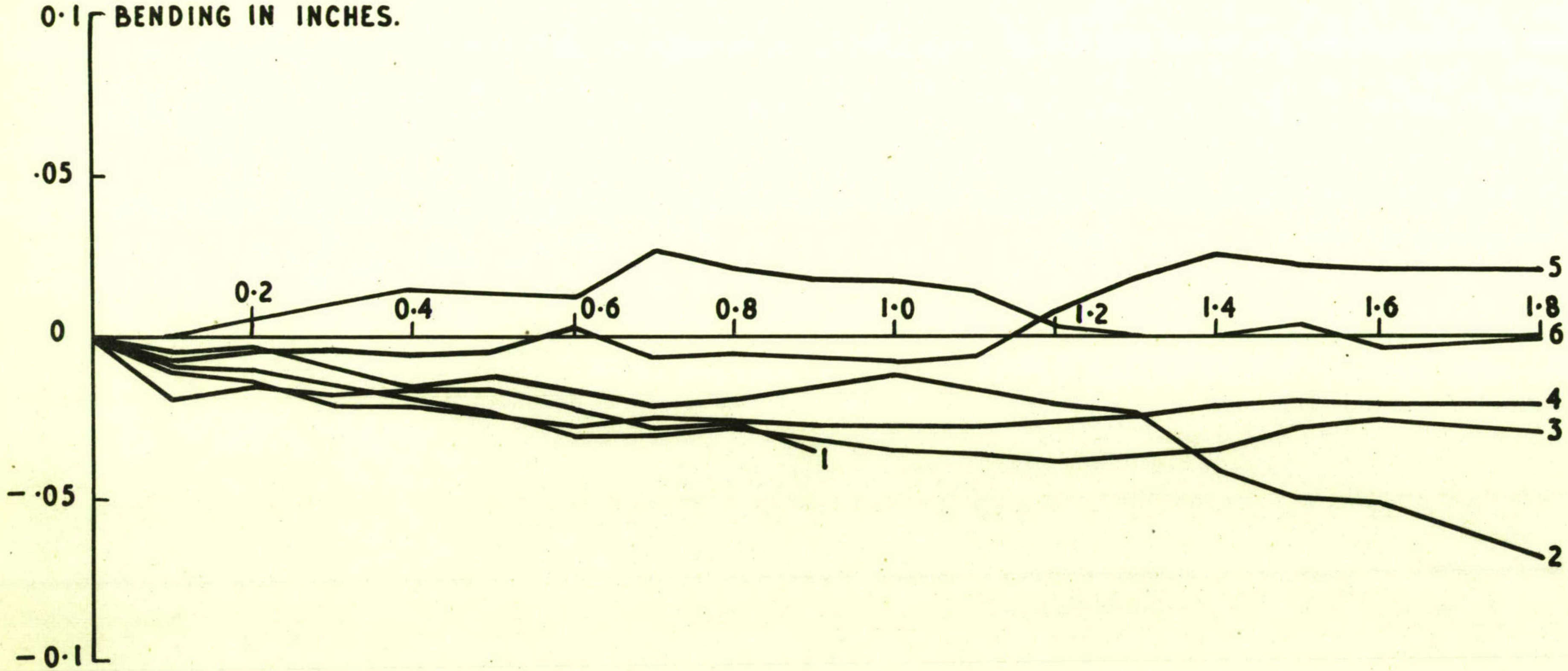

FIG. 23. 3" No. 1 TAPERED FB2 STAR IN F 488/649 HEAD. END OF CHARGE SEALED. WELDED SEAL AT HEAD END OF TUBE. REFRACTORY LINING INTACT.

FIG. 23.

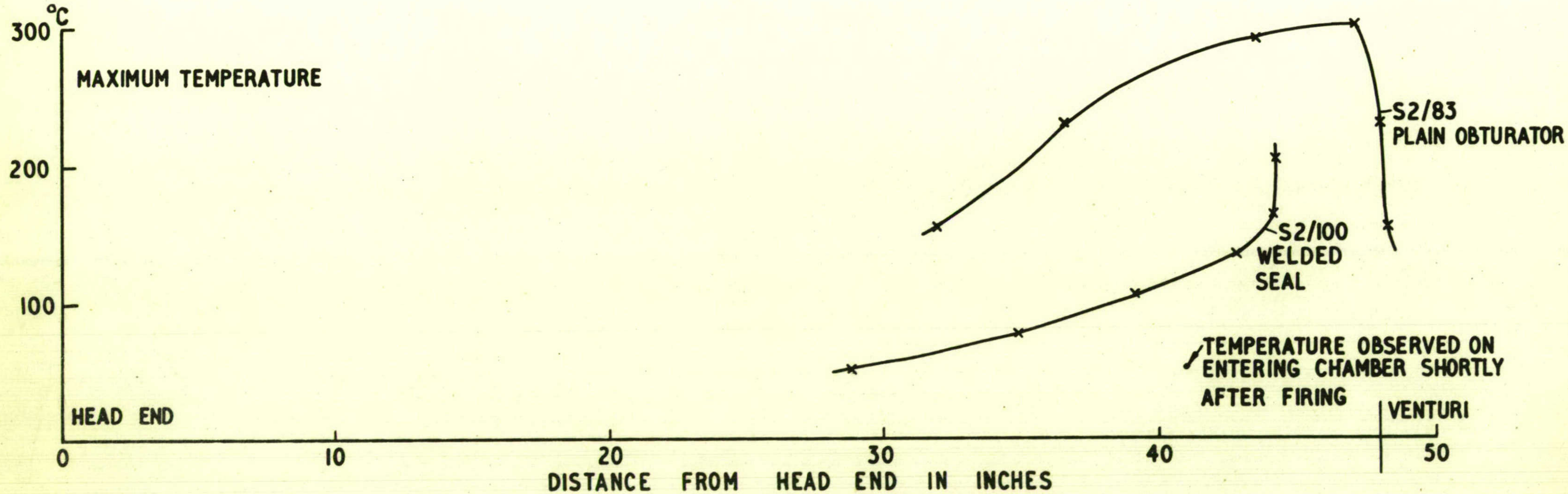

FIG.24. COMPARISON OF MAXIMUM TEMPERATURES (AS INDICATED BY "TEMPILAQ" PAINTS) OBTAINED ON 3" No 1. ROCKET WITH SEALED HEAD STAR CENTRE CHARGES USING A PLAIN OBTURATOR AND A WELDED SEAL.

FIG. 25.

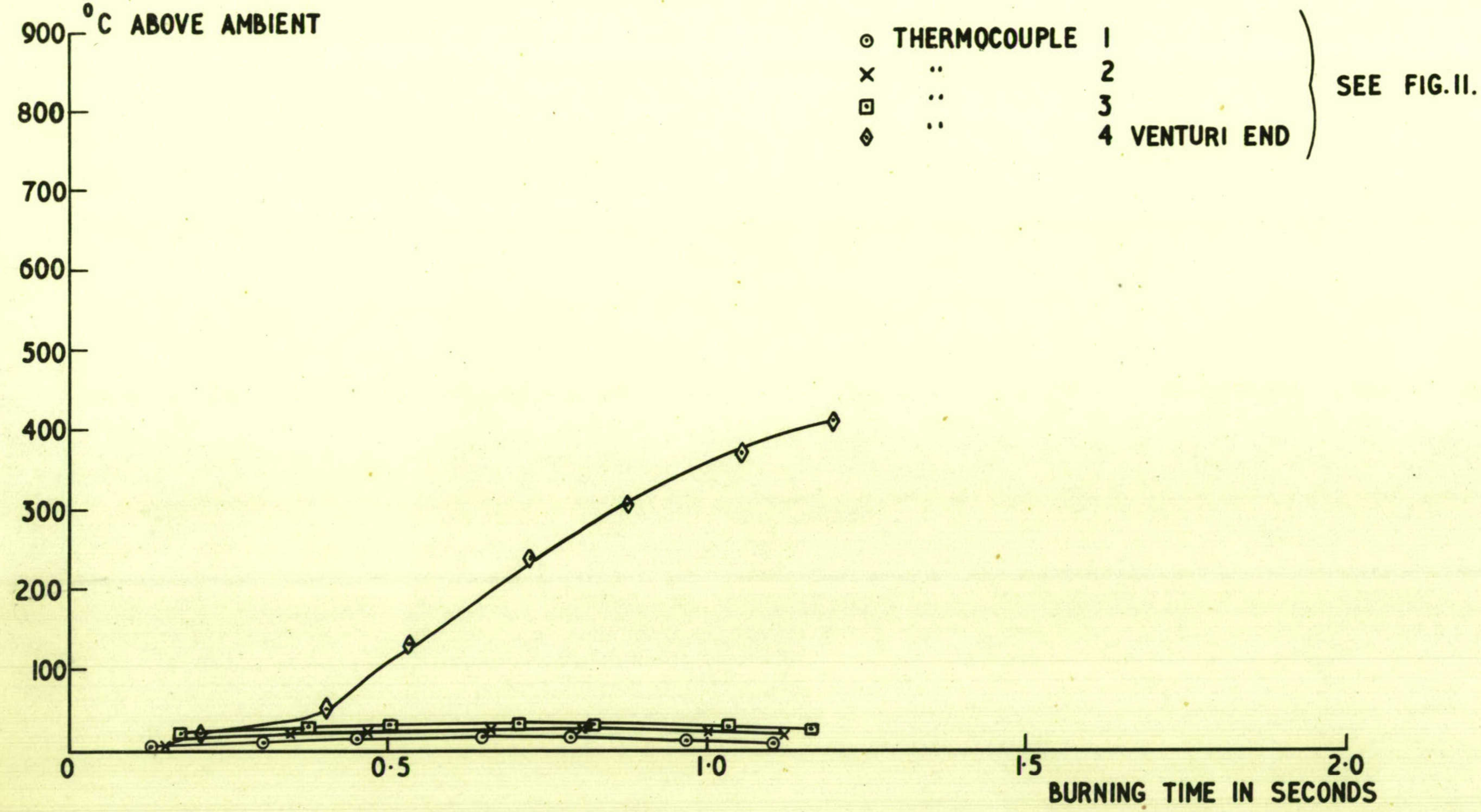

FIG. 25. 3" ROCKET CHARGE F488/649 FB 2 STAR WITH STEPPED CONDUIT. HEAD END OF CHARGE SEALED. HEAD END OF TUBE SEALED BY WELD.

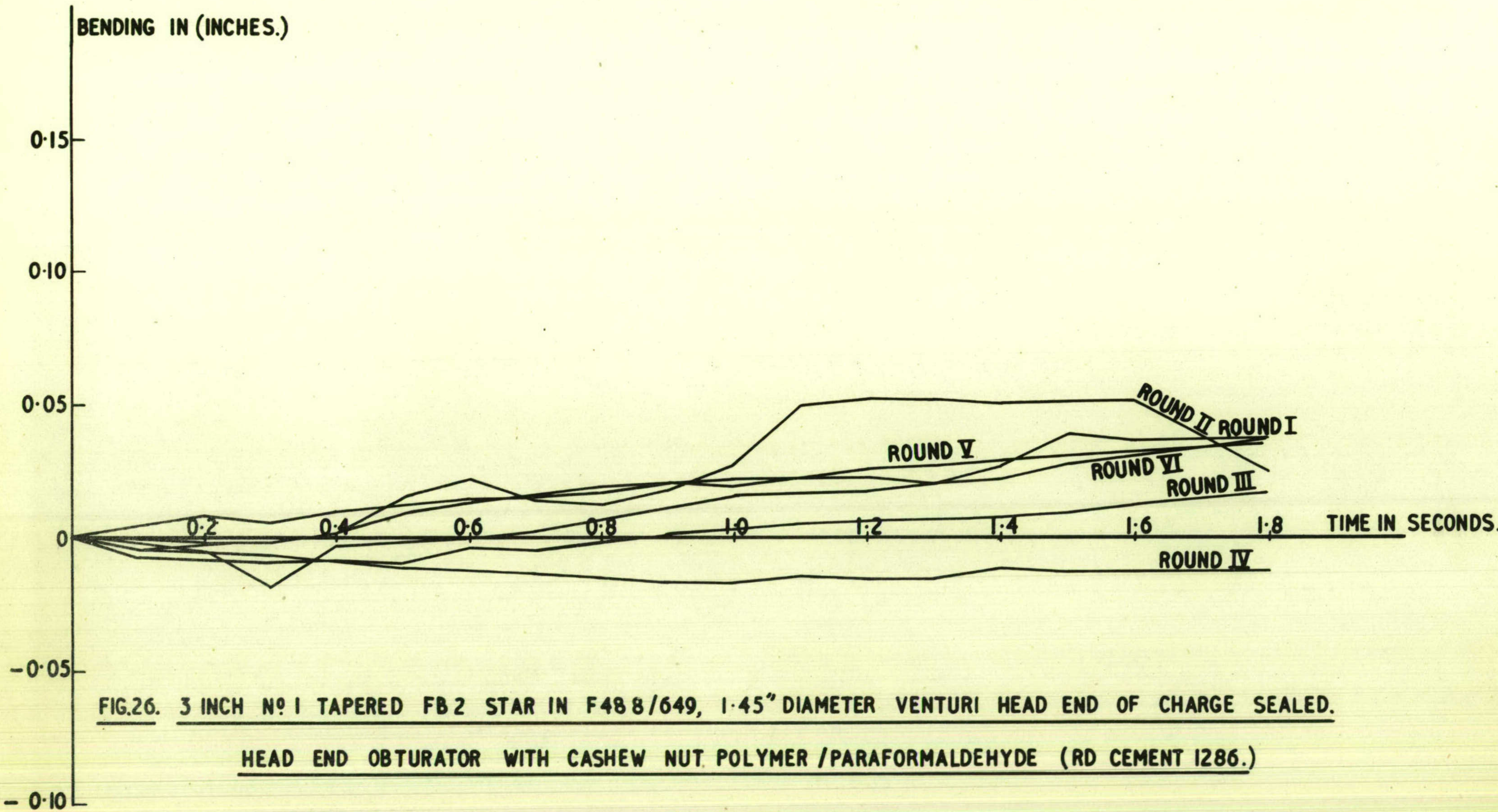

FIG.26. 3 INCH Nº 1 TAPERED FB 2 STAR IN F48 8/649, 1·45" DIAMETER VENTURI HEAD END OF CHARGE SEALED.

HEAD END OBTURATOR WITH CASHEW NUT POLYMER / PARAFORMALDEHYDE (RD CEMENT 1286.)

FIG. 26.

FIG. 27.

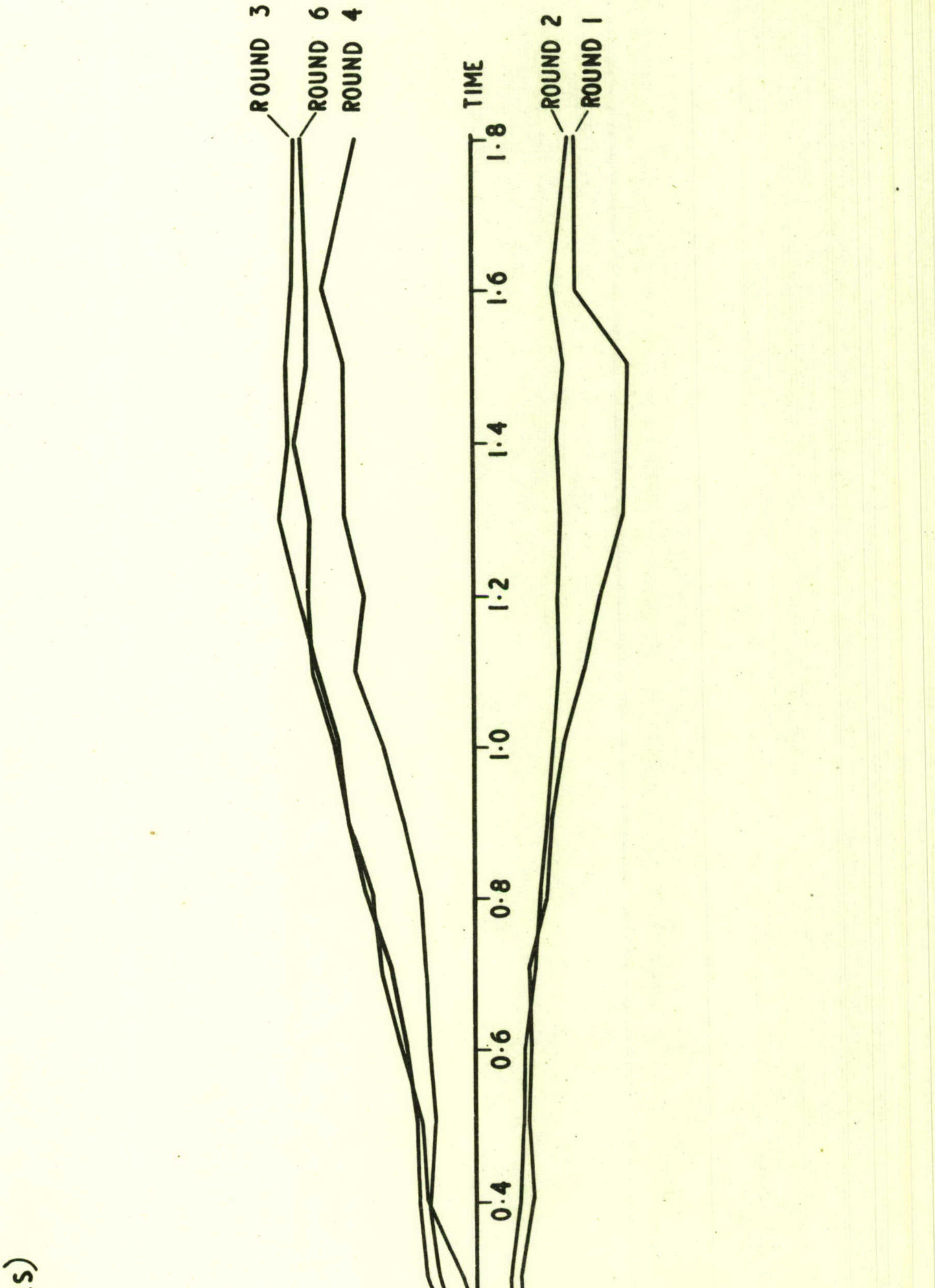

FIG. 27. 3 INCH No. 1 2·75-0·75 SUK DRILLED TUBULAR. ·008 REFRACTORY LINING INTACT. ·18 INCH THICK TUBES.

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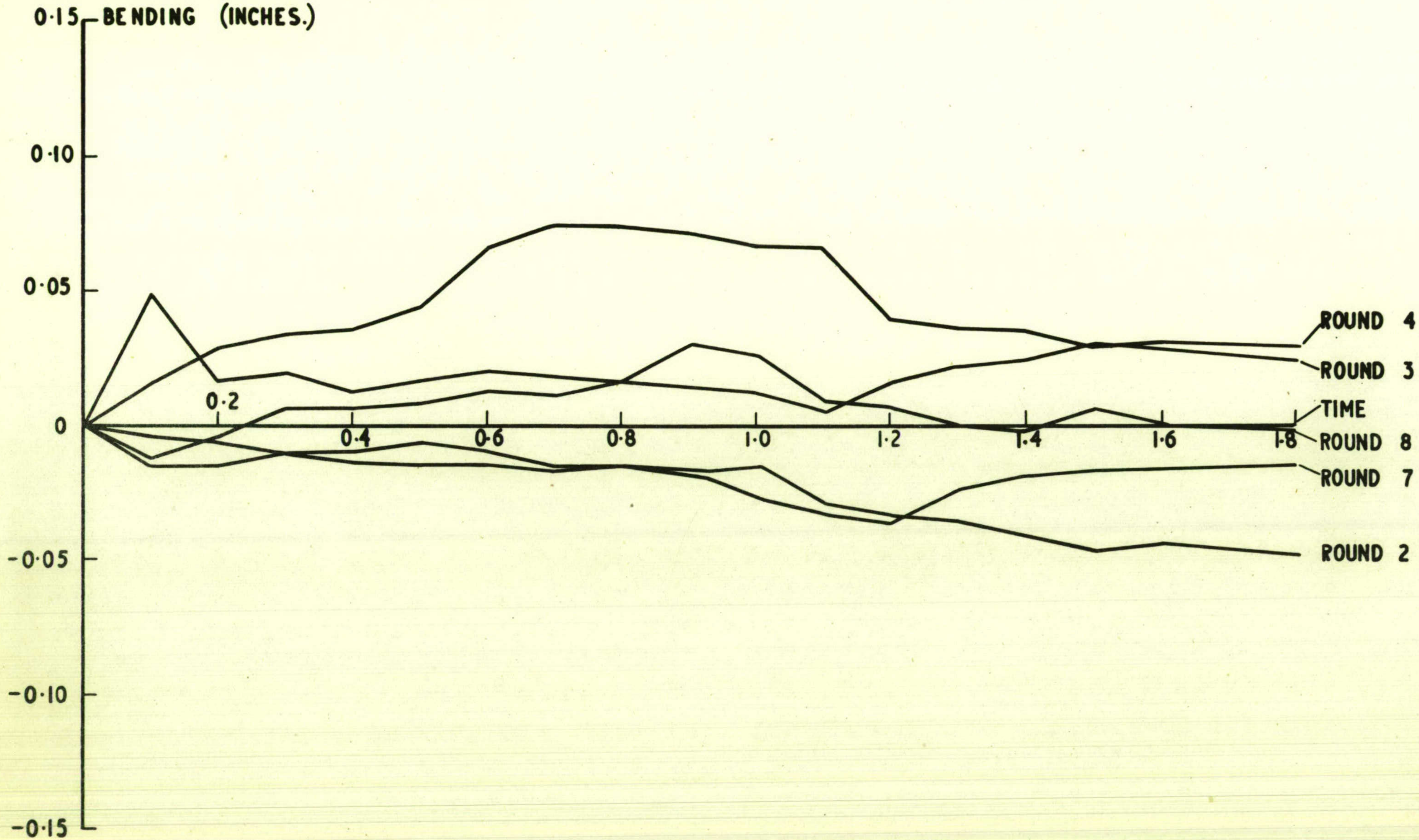

FIG. 28. 3 INCH No.1. 2·75–0·75 S.U.K. DRILLED TUBULAR. ·008 REFRACTORY LINING REMOVED. ·18 INCH THICK TUBES.

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FIG. 29.

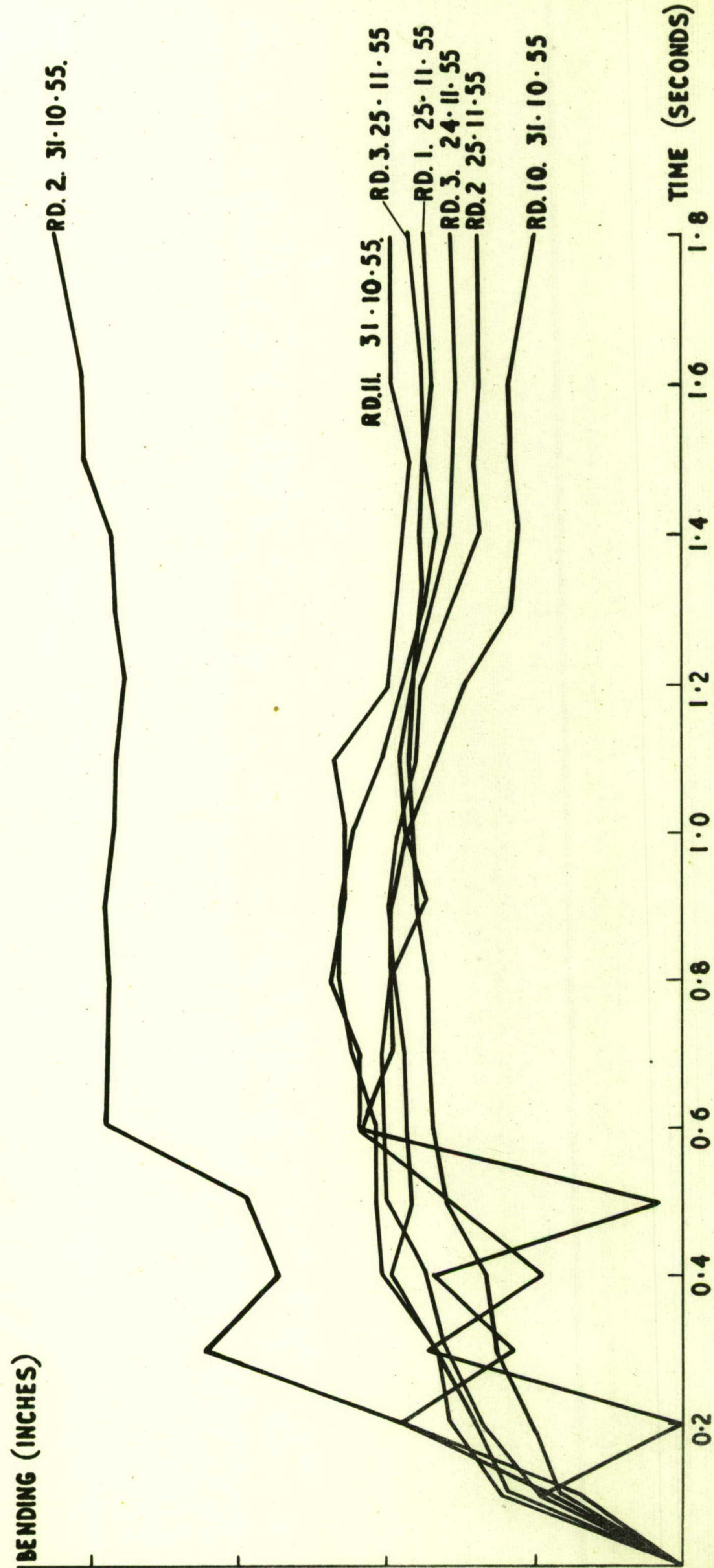

FIG. 29. 3 INCH No. 1 2·75—0·75 S.U.K. DRILLED TUBULAR—OFFSET 0·1 INCH FROM AXIS.

·008 REFRACTORY LINING REMOVED. 0·18 INCH WALL TUBE.

FIG. 30.

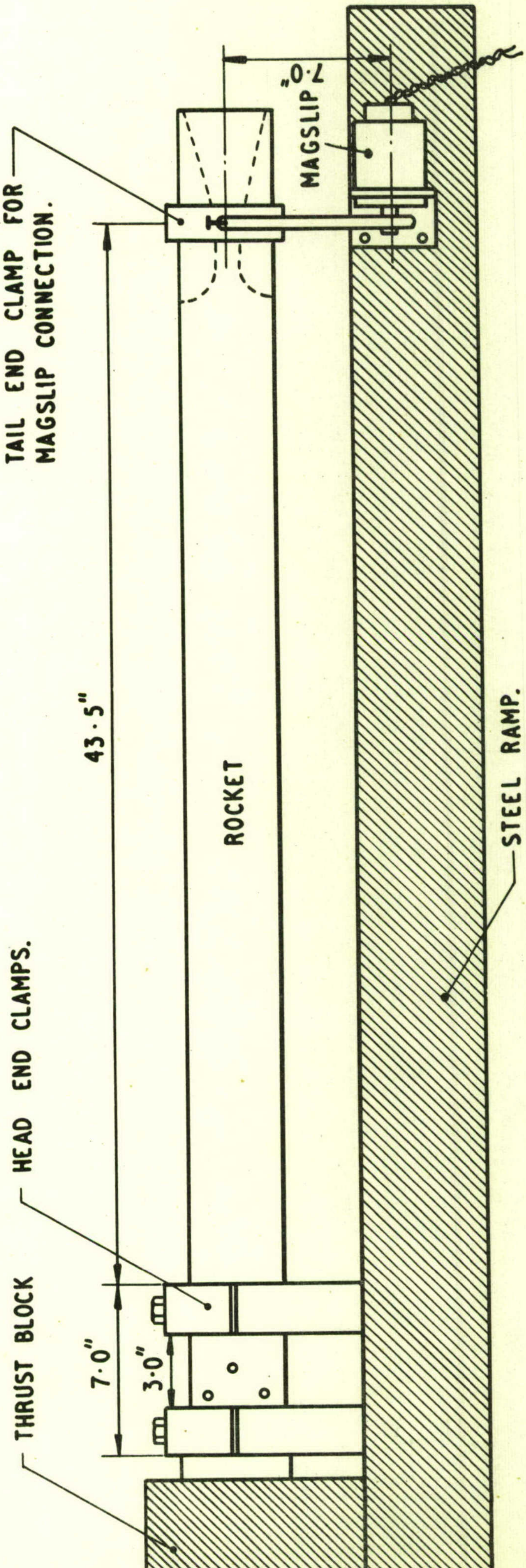

FIG. 30. ARRANGEMENT OF MAGSLIP AND CLAMPS USED IN BENDING MEASUREMENT.

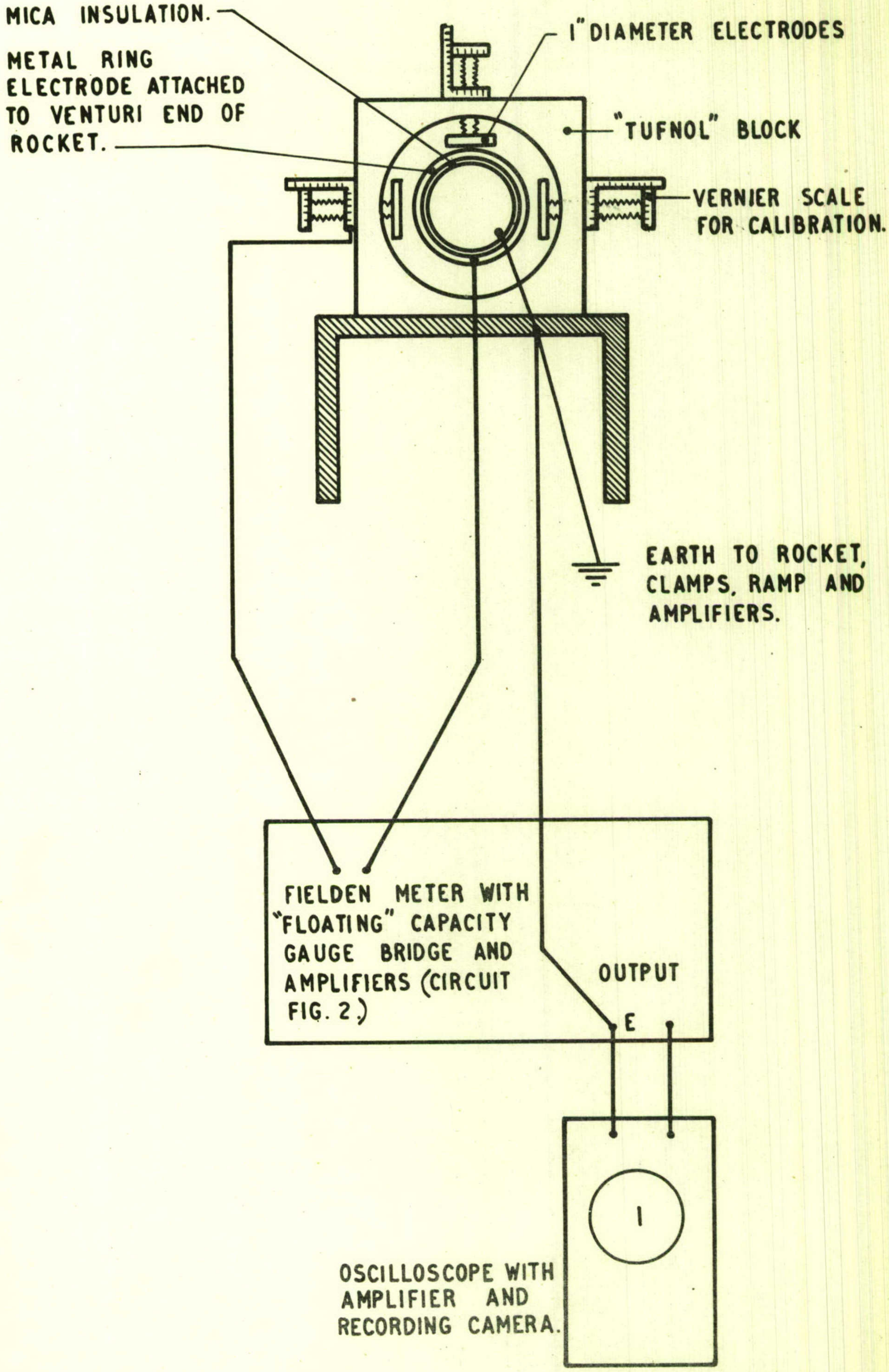

FIG. 31. MEASUREMENT OF BENDING OF ROCKET DURING STATIC FIRING BY MEANS OF "FLOATING" CAPACITY GAUGE.

FIG. 32.

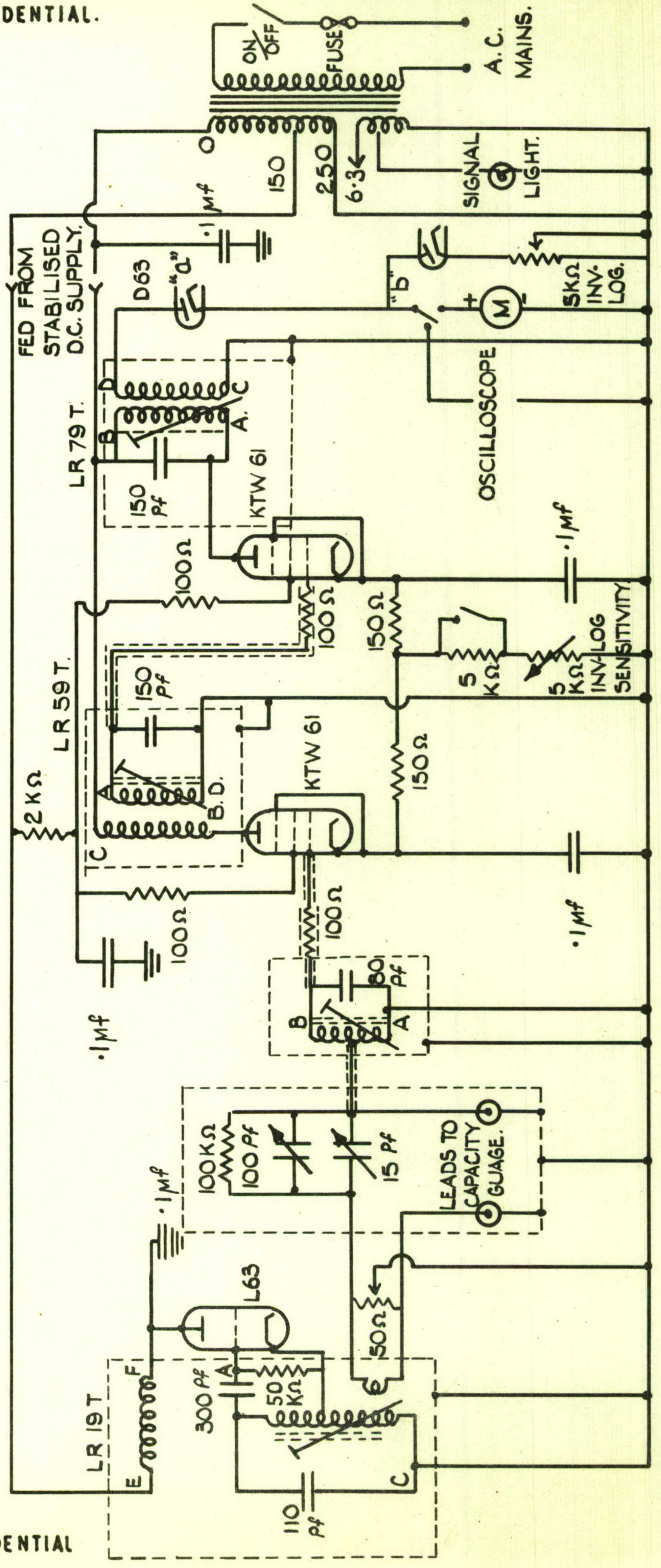

FIG. 32. MODIFIED FIELDEN UNIT WITH "FLOATING" CAPACITY GAUGE.

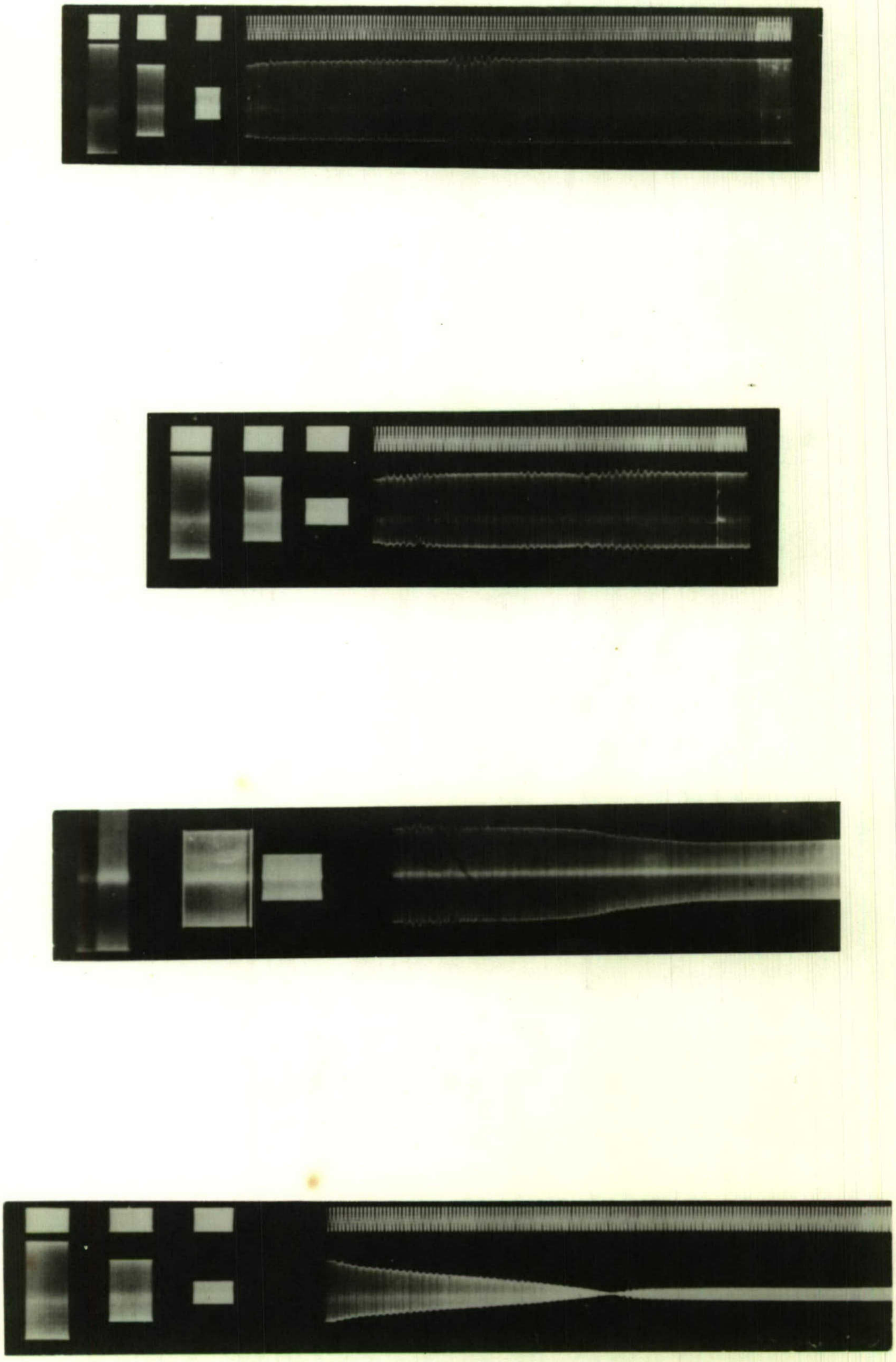

FIG. 33. TYPICAL RECORDS USING MAGSLIP TECHNIQUE.

UPPER TRACE 50 C.P.S. TIMING.

LOWER TRACE 0·25 INCH CALIBRATION STEPS (FIRING RECORD).

FIG. 34.

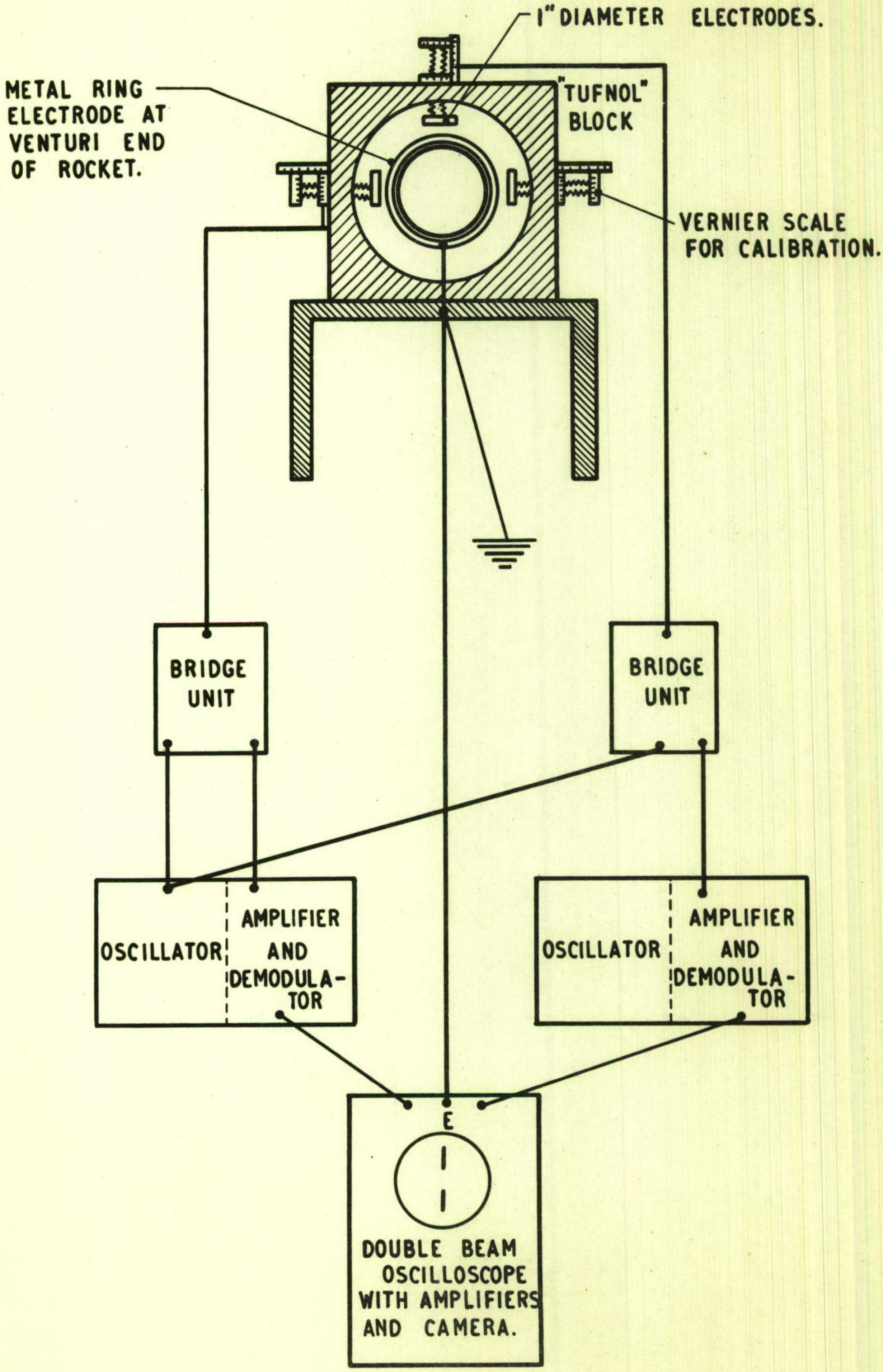

FIG. 34. MEASUREMENT OF BENDING OF ROCKET DURING STATIC FIRING BY MEANS OF EARTHED CAPACITY GAUGE.

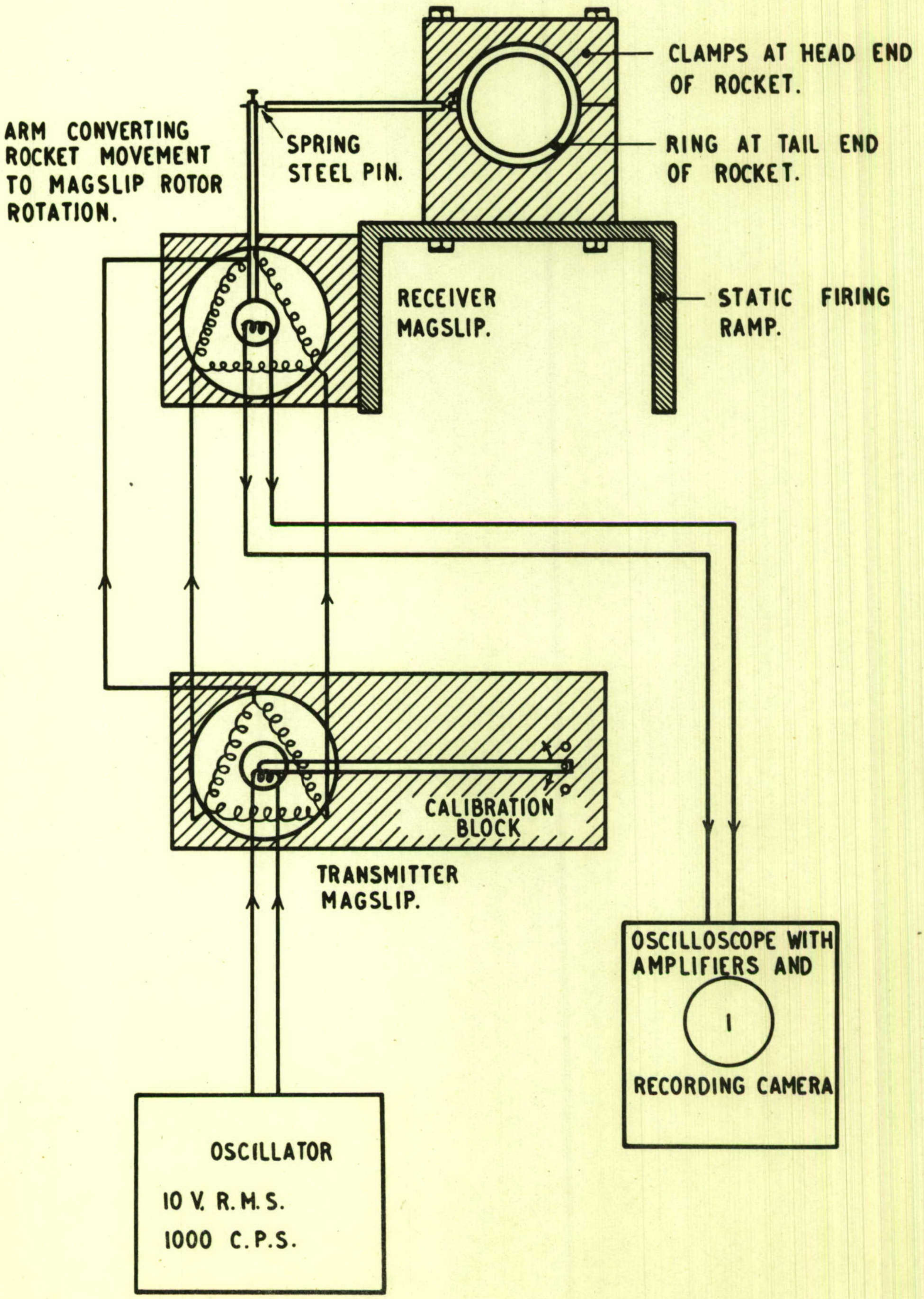

FIG. 35. MEASUREMENT OF BENDING OF ROCKET DURING STATIC FIRING BY MEANS OF MAGSLIPS.

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